Review for MTG4302

(1) Definitions (and their negations).
   (a) A topology on a space X is a family of subsets of X, say τ, such that
   (b) Examples of topologies: discrete, indiscrete (aka trivial), finite complement, upper limit, lower limit
   (c) Let σ, τ be topologies on X, we say σ is finer (resp. coarser) than τ if
   (d) A basis for a topology on X is a family of subsets of X, say $\mathscr{B}$, such that
   (e) An open set is
   (f) The subset $A \subseteq X$ is called closed if
   (g) The subset $A \subseteq X$ is called clopen if
   (h) The space X is called connected if ; otherwise it is disconnected.
   (i) A topological space X is said to be Hausdorff if
   (j) Given $A \subseteq X$, we define the interior of A as
   (k) Given $A \subseteq X$, we define the closure of A as
   (l) Given $A \subseteq X$, we say A is a dense subset of X if
   (m) A function $d : X \times X \to \mathbb{R}^+$ is called a metric if (looking for the 3 properties)
   (n) Given a metric space (X, d) for each $x \in X$, $r > 0$ we define the open ball centered around x of radius r as $N_r(x) =$
   (o) We say the space X satisfies the T_0-separation axiom if
   (p) We say the space X satisfies the T_1-separation axiom if
   (q) We say the space X satisfies the T_2-separation axiom if
   (r) Given an example of a space that satisfies T_0 but T_1.
   (s) Given an example of a space that satisfies T_1 but T_2.

(2) Proofs
   (a) Given a metric space (X, d), prove that the collection $\{N_r(x) : r > 0, x \in X\}$ is a basis for a topology on X.
   (b) Prove a metric space is Hausdorff.
   (c) Prove that the closure of a set A if the set of $x \in X$ such that whenever $x \in O$ and O is open then $O \cap A \neq \emptyset$. (See Theorem 2.5)
   (d) Be able to prove the different parts of Theorem 2.2.
   (e) Be able to prove the different parts of Theorem 2.6.
   (f) Be able to compare different topologies on a fixed set.
   (g) Prove that a certain collection of subsets of X forms a basis for a topology on X.
   (h) prove that for any collection of subsets $\{O_i\}$ and set V,

$$(\bigcap_i O_i) \cup V = \bigcap_i ((O_i \cup V) \text{ and } (\bigcup_i O_i) \cap V = \bigcup_i ((O_i \cap V)$$