$G = \left\{ \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \middle| \theta \in \mathbb{R} \right\}$ $\quad G \leq \underline{SL_2(\mathbb{R})}$

$H = \left\{ \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \middle| \theta = 2\pi q \text{ some } q \in \mathbb{Q} \right\}$

✓ not empty

$\forall a,b \in H,\ ab \in H$

$g\pi$

$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} = \begin{pmatrix} \underline{\cos\theta\cos\alpha - \sin\theta\sin\alpha} & -\cos\theta\sin\alpha - \sin\theta\cos\alpha \\ \sin\theta\cos\alpha + \cos\theta\sin\alpha & -\sin\theta\sin\alpha + \cos\theta\cos\alpha \end{pmatrix}$ $= \cancel{\cos\theta}\ \sin(\theta+\alpha) +$

$\underline{\text{why}}?$ trig ident.

$= \begin{pmatrix} \underline{\cos(\theta+\alpha)} & -\sin(\theta+\alpha) \\ \sin(\theta+\alpha) & \cos(\theta+\alpha) \end{pmatrix}$

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{\det A}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$\det \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} = \cos^2\theta + \sin^2\theta = 1$

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$\forall h \in H,\ h^{-1} \in H$

$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}^{-1} = \frac{1}{1}\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{pmatrix} \in G$

cos is even
sin odd

f even
$f(-x) = f(x)$

f odd
$f(-x) = -f(x)$

what's special about H?

1) $|H| = +\infty$

2) $\forall h \in H,\ |h| < \infty$