MAS 4301 – $SL(2, \mathbb{F}_3)$

We are going to find all the subgroups of $SL(2, \mathbb{F}_3)$. We use the fact that $|SL(2, \mathbb{F}_3)| = 24$ and so a subgroup of must have size 1,2,3,4,6,8,12, or 24.

We first find the cyclic subgroups and orders of elements. We let I_2 be the identity matrix. Observe that $|2I_2|$=2

1. $< I_2 >= \{I_2\}$
2. $< 2I_2 >= \{2I_2, I_2\}$.

3. $< \begin{vmatrix}1&1\\0&1\end{vmatrix} >= \{\begin{vmatrix}1&1\\0&1\end{vmatrix}, \begin{vmatrix}1&2\\0&1\end{vmatrix}, I_2\} =< \begin{vmatrix}1&2\\0&1\end{vmatrix} >$
4. $< \begin{vmatrix}1&0\\1&1\end{vmatrix} >= \{\begin{vmatrix}1&0\\1&1\end{vmatrix}, \begin{vmatrix}1&0\\2&1\end{vmatrix}, I_2\} =< \begin{vmatrix}1&0\\2&1\end{vmatrix} >$
5. $< \begin{vmatrix}2&0\\2&2\end{vmatrix} >= \{\begin{vmatrix}2&0\\2&2\end{vmatrix}, \begin{vmatrix}1&0\\2&1\end{vmatrix}, \begin{vmatrix}2&0\\0&2\end{vmatrix}, \begin{vmatrix}1&0\\1&1\end{vmatrix}, \begin{vmatrix}2&0\\1&2\end{vmatrix}, I_2\} =< \begin{vmatrix}2&0\\1&2\end{vmatrix} >$
6. $< \begin{vmatrix}2&2\\0&2\end{vmatrix} >= \{\begin{vmatrix}2&2\\0&2\end{vmatrix}, \begin{vmatrix}1&2\\0&1\end{vmatrix}, \begin{vmatrix}2&0\\0&2\end{vmatrix}, \begin{vmatrix}1&1\\0&1\end{vmatrix}, \begin{vmatrix}2&1\\0&2\end{vmatrix}, I_2\} =< \begin{vmatrix}2&1\\0&2\end{vmatrix} >$

7. $< \begin{vmatrix}2&1\\2&0\end{vmatrix} >= \{\begin{vmatrix}2&1\\2&0\end{vmatrix}, \begin{vmatrix}0&2\\1&2\end{vmatrix}, I_2\} =< \begin{vmatrix}0&2\\1&2\end{vmatrix} >$
8. $< \begin{vmatrix}0&1\\2&2\end{vmatrix} >= \{\begin{vmatrix}0&1\\2&2\end{vmatrix}, \begin{vmatrix}2&2\\1&0\end{vmatrix}, I_2\} =< \begin{vmatrix}2&2\\1&0\end{vmatrix} >$
9. $< \begin{vmatrix}0&1\\2&0\end{vmatrix} >= \{\begin{vmatrix}0&1\\2&0\end{vmatrix}, \begin{vmatrix}2&0\\0&2\end{vmatrix}, \begin{vmatrix}0&2\\1&0\end{vmatrix}, I_2\} =< \begin{vmatrix}0&2\\1&0\end{vmatrix} >$
10. $< \begin{vmatrix}1&1\\2&0\end{vmatrix} >= \{\begin{vmatrix}1&1\\2&0\end{vmatrix}, \begin{vmatrix}0&1\\2&2\end{vmatrix}, \begin{vmatrix}2&0\\0&2\end{vmatrix}, \begin{vmatrix}2&2\\1&0\end{vmatrix}, \begin{vmatrix}0&2\\1&1\end{vmatrix}, I_2\} =< \begin{vmatrix}0&2\\1&1\end{vmatrix} >$
11. $< \begin{vmatrix}0&1\\2&1\end{vmatrix} >= \{\begin{vmatrix}0&1\\2&1\end{vmatrix}, \begin{vmatrix}2&1\\2&0\end{vmatrix}, \begin{vmatrix}2&0\\0&2\end{vmatrix}, \begin{vmatrix}0&2\\1&2\end{vmatrix}, \begin{vmatrix}1&2\\1&0\end{vmatrix}, I_2\} =< \begin{vmatrix}1&2\\1&0\end{vmatrix} >$

12. $< \begin{vmatrix}1&2\\2&2\end{vmatrix} >= \{\begin{vmatrix}1&2\\2&2\end{vmatrix}, \begin{vmatrix}2&0\\0&2\end{vmatrix}, \begin{vmatrix}2&1\\1&1\end{vmatrix}, I_2\} =< \begin{vmatrix}2&1\\1&1\end{vmatrix} >$
13. $< \begin{vmatrix}1&1\\1&2\end{vmatrix} >= \{\begin{vmatrix}1&1\\1&2\end{vmatrix}, \begin{vmatrix}2&0\\0&2\end{vmatrix}, \begin{vmatrix}2&2\\2&1\end{vmatrix}, I_2\} =< \begin{vmatrix}2&2\\2&1\end{vmatrix} >$

14. G