MAS 4301 – Operations

Let's have some practice on operations.

Example 0.1. We define an operation on $A = \mathbb{R}^2$. First, recall that $\mathbb{R}^2$ is the set of ordered pairs of the form (a, b) for some $a, b \in \mathbb{R}$. A binary operation on R^2 must ake two ordered pairs and output a single ordered pair. Let $(a_1, a_2), (b_1, b_2) \in \mathbb{R}^2$ and define

$$(a_1, a_2)\triangle(b_1, b_2) = (a_1 + b_1, a_2 b_2).$$

(1) Compute $(1, 3)\triangle(2, -3)$.

(2) We want to show this is commutative. We know what $(a_1, a_2)\triangle(b_1, b_2)$ equals so calculate

$$(b_1, b_2)\triangle(a_1, a_2)$$

(3) Let's determine whether there is a $\triangle$-identity. We suppose that (e_1, e_2) is a $\triangle$-identity and then calculate two things and set them equal to each other. First, compute

$$(e_1, e_2)\triangle(a_1, a_2) =$$

and since it is an identity we set that equal to (a_1, a_2). So what is the $\triangle$-identity?

Example 0.2. On the set $A = \mathbb{R} \setminus \{0\}$ of non-zero real numbers define

$$r * s = r \cdot s^{\operatorname{sgn}(r)}.$$

Recall that $\operatorname{sgn}(x) = \begin{cases} 1, & \text{if } x > 0 \\ -1, & \text{if } x < 0. \end{cases}$

(1) First, compute $(-1) * 2$. Then compute $2 * (-1)$. What does this tell you about the operation?

(2) Next, we are going to compute both $(r*s)*t$ and separately and then compare. We probably should prove this using cases.

Case 1: If $r, s > 0$, then $(r * s) * t = (rs) * t = rst$, while $r * (s * t) = r * (st) = rst$. Yes!!

Case 2: If $r > 0, s < 0$, then $(r * s) * t = (rs) * t = \frac{rs}{t}$. What is $r * (s * t)$? First use that $s * t = \frac{s}{t}$.

Now you do the last two cases:

Case 3:

Case 4: