MAS 4301 – Matrix Multiplication

We want you to work out the proof that matrix multiplication is associative.

Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $B = \begin{pmatrix} e & f \\ g & h \end{pmatrix}$, $C = \begin{pmatrix} x & y \\ z & w \end{pmatrix}$, We aim to show that $(A \otimes B) \otimes C = A \otimes (B \otimes C)$.

So we need to computer $A \otimes B$. But we did that in the definition

$$A \otimes B = \begin{pmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{pmatrix}.$$

We now have to multiply

$$(A \otimes B) \otimes C = \begin{pmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{pmatrix} \otimes \begin{pmatrix} x & y \\ z & w \end{pmatrix} =$$

Next, we need

$$B \otimes C =$$

Then compute

$$A \otimes (B \otimes C) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \otimes \qquad\qquad =$$

Conclusion?

Next, show that the matrix $I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is a $\otimes$-identity on $M_2(\mathbb{R})$.

$$A \otimes I_2 =$$

$$I_2 \otimes A =$$

Example 0.1. Pick at random two matrices D and E. (Don't use I_2.) Do they commute?