MAS 4301 – Functions

In class we discussed the collection of functions from a fixed set S into the real numbers. The appropriate set-theoretic notation would be $\mathbb{R}^S$ but I have found that this notation is not so suggestive. So let's denote this set by $\mathcal{F}(S, \mathbb{R})$. Here the $\mathcal{F}$ can remind you of functions, and then S goes first since it is the domain, and then $\mathbb{R}$ since it is the co-domain.

In class we defined an operation on $\mathcal{F}(S, \mathbb{R})$ as follows: for $f, g \in \mathcal{F}(S, \mathbb{R})$, we defined $f \oplus g$ to be the function that sends each $s \in S$ to the real number $f(s) + g(s)$. We showed that the operation is associative and commutative. We also showed that there is a $\oplus$-identity: the function $\bar{0}$ which sends everything in S to $0 \in \mathbb{R}$.

$$\bar{0} \oplus f = f.$$

We also showed that each function $f \in \mathcal{F}(S, \mathbb{R})$ has an $\oplus$-inverse. It is the function that sends each $s \mapsto -f(s)$. Suggestively, we denote this function by $-f$. It follows that

$$f \oplus -f = \bar{0}.$$

Remark 0.1. The big idea in dealing with functions is that to show two functions $f, g \in \mathcal{F}(S, \mathbb{R})$ are equal, you have to show that **for all** $s \in S$**,** $f(s) = g(s)$.

Example 0.2. Let $T = \{0, 1\}$ and let $f, g, h \in \mathcal{F}(T, \mathbb{R})$ be defined by $f(t) = 2t+1$, $g(t) = 1+4t-2t^2$, and $h(t) = 5^t + 1$.

i) Show that $f = g$.

ii) Show that $f \oplus g = h$.

iii) Is f injective?

iv) Let $j \in \mathcal{F}(T, \mathbb{R})$ be defined by $j(t) = \sin(\pi t)$. True or False: $j = \bar{0}$.

v) Is f surjective? If no, is there any $k \in \mathcal{F}(T, \mathbb{R})$ that is surjective.

vi) If we take $T' = \{0, 1, 2\}$ are i), ii), iii) and iv) still true?