MAS 4301 – Examples of Groups

1. The set of reals numbers $\mathbb{R}$ under addition. The group of integers and the group or rational numbers are subgroups of $\mathbb{R}$ under addition.

2. The set of non-zero reals, $\mathbb{R}^*$ is a group under multiplication with 1 as its identity and inverses are the reciprocals. The set of nonzero rationals is a subgroup.

3. For any $1 < n \in \mathbb{N}$, $Z_n = \{0, 1, 2, \ldots, n-1\}$ is a group under modular addition. The identity is 0 and the inverse of $a \in \mathbb{Z}_n$ is $n - a$.

4. For any $1 < n \in \mathbb{N}$, $U(n) = \{k \in Z_n : gcd(n, k) = 1\}$ under multiplication with 1 as its identity. To find the inverse of a given element solve the equation $1 = ny + xk$ and then take $x = x' \mod n$.

5. For a set A, the group of permutation on A under composition is denoted by S_A and called the symmetric group on A. The identity of S_A is the identity function. Each element has a cycle decomposition. $|S_n| = n!$

6. A_n is the subgroup of S_n consisting of the even permutations. $|A_n| = \frac{n!}{2}$.

7. For any group $(G, *, e)$ and any $g \in G$, the cyclic subgroup $< g >$ is a subgroup of G.

8. The set $M_2(\mathbb{R})$ of 2×2 matrices with real coefficients under addition.

9. For a field $\mathbb{F}$, The set of invertible matrices in $M_n(\mathbb{F})$: $GL_n(\mathbb{F}) = \{A \in M_n(\mathbb{F}) : \det(A) \neq 0\}$ and is called the **general linear group**. Those matrices whose determinant equal 1 is a subgroup called the **special liner group** $SL_n(\mathbb{F})$.

10. The quaternion group $Q_8 = \{\pm 1, \pm i, \pm j, \pm k\}$.

11. The dihedral group D_n which is the set of rigid transformations on a regular n-gon. $|D_n| = 2n$ and can be viewed as a subgroup of S_n by labeling the vertices of the n-gon.