Name: ____

Exam 1– MAS 4301H Fall 2022

Remark: Throughout the test n denotes a positive integer greater than 1. We consider the group $\mathbb{Z}_n = \{0, 1, \ldots, n-1\}$ under addition.

1. Definitions:
   (a) State the definition of a group.

   (b) What does it mean for the group $(G, \cdot, e)$ to be an *abelian* group?

   (c) What does it mean for the group $(G, \cdot, e)$ to be a *cyclic* group?

   (d) The center of a group is

   $$Z(G) = \{ \qquad\qquad\qquad\qquad\qquad\qquad \}$$

   (e) List the elements of $U(21) = \{ \qquad\qquad\qquad\qquad\qquad\qquad \}$

   (f) For a group G and $g \in G$, define the *order of* g: $|g|$

   (g) Let A be a set. What is a permutation on A?

2. Given an example for each of the following.
   i. A cyclic group:

   ii. An abelian group which is not cyclic:

   iii. A non-abelian group:

   iv. Find the inverse of 5 in $U(11)$.

3. Let $a \in Z_n$. What is the order of $|a|$? Use this find the order of 3 in Z_{111}?

4. What is the order of the dihedral group D_n?

5. What are the possible orders of elements in S_6?

6. In S_6, let $\sigma = (1\ 3\ 5)(2\ 4)$ and $\tau = (2\ 6)$.

   i. What is $|\sigma|$?

   ii. Compute $\tau \circ \sigma$.

   iii. Give the cycle decomposition of σ^{-1}.

7. Let $(G, e, \cdot)$ be a group. Prove that if for all $a \in G$, $a^2 = e$, then G is abelian.

8. Let $(G, \cdot, e)$ be a group and $a, g \in G$. Prove that $|g| = |aga^{-1}|$.

9. Let G be an abelian group. Show that the elements of finite order in G form a subgroup.

10. Let G be a group and define a map $\lambda_g : G \to G$ by $\lambda_g(a) = ga$. Prove that λ_g is a permutation of G.