

$$G = \langle x, y \mid x^3 = e = y^4, xy = yx^2 \rangle$$

e	x	x^2
y	xy	x^2y
y^2	xy^2	x^2y^2
y^3	xy^3	x^2y^3

$$|G| = 12$$

$$xy = yx^2$$

$$x^2y = yx$$

$$xy^2 = yx^2y = y^2x \quad (y^2 \in Z(G))$$

$$x^2y^2 = y^2x^2$$

$$xy^3 = yx^2y^2 = y^3x^2$$

$$x^2y^3 = y^3x$$

$$\langle xy \rangle = \{xy, y^2, xy^3, e\} = \langle xy^3 \rangle$$

$$\langle x^2y \rangle = \{x^2y, y^2, x^2y^3, e\} = \langle x^2y^3 \rangle$$

$$\langle xy^2 \rangle = \{xy^2, x^2, y^2, x, x^2y^2, e\} = \langle x^2y^2 \rangle$$

$$\langle x \rangle = \{x, x^2, e\} = \langle x^2 \rangle$$

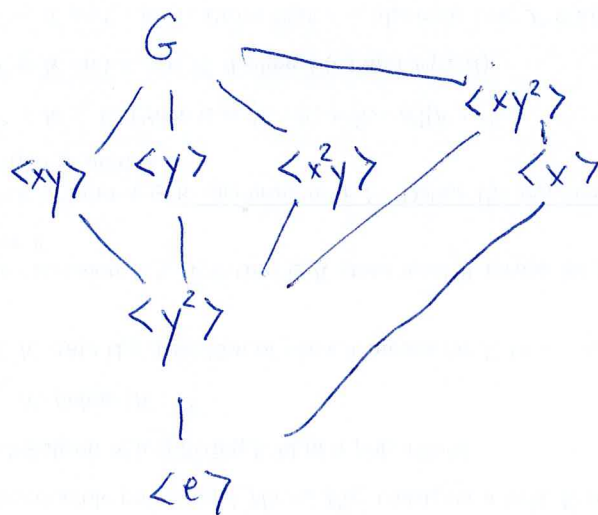
$$\langle y \rangle = \{y, y^2, y^3, e\} = \langle y^3 \rangle$$

$$\langle y^2 \rangle = \{y^2, e\}$$

x, y^2 commute

$$\langle x \rangle \cap \langle y^2 \rangle = \{e\}$$

$$\Rightarrow |\langle xy^2 \rangle| = \text{lcm}(3, 2) = 6$$



Lagrange's

$$[G : \langle xy \rangle] = 3$$

$$[G : \langle y \rangle] = 3$$

$$[G : \langle x^2y \rangle] = 3$$

$$[G : \langle xy^2 \rangle] = 2$$