$|G| = 20$ $\quad G = \langle x,y \mid x^5 = y^4 = 1,\ xy = yx^4 \rangle$

$\langle xy \rangle = \{xy, y^2, xy^3, \cancel{y}e\} = \langle xy^3 \rangle$

$x \leftrightarrow y^2$ commute so $|xy^2| = \text{lcm}(|x|, |y^2|) = 10$

$\langle xy^2 \rangle = \{xy^2, x^2, x^3y^2, x^4, \cancel{x}y^2, x, x^2y^2, x^3, x^4y^2, e\}$

$= \langle x^3y^2 \rangle = \langle x^2y^2 \rangle = \langle x^4y^2 \rangle$

$\langle x \rangle = \langle x^2 \rangle = \langle x^3 \rangle = \langle x^4 \rangle$

$\langle y \rangle = \langle y^3 \rangle$

$\langle y^2 \rangle$

$\langle x^2y \rangle = \{x^2y, y^2, x^2y^3, e\} = \langle x^2y^3 \rangle$

$\langle x^3y \rangle = \{x^3y, y^2, x^3y^3, e\} = \langle x^3y^3 \rangle$

$\langle x^4y \rangle = \{x^4y, y^2, x^4y^3, e\} = \langle x^4y^3 \rangle$

done w/ cyclic

$y^{-1}xy = x^4$

$y^{-1}x^2y = (y^{-1}xy)^2 = x^3$

$y^{-1}x^3y = x^2$

$y^{-1}x^4y =$

$$\boxed{\begin{array}{l} xy = yx^4 \\ x^2y = yx^3 \\ x^3y = yx^2 \\ x^4y = yx \end{array}}$$

~~trivial center~~?

$y^2 \in Z(G)$?

$xy^2 = yx^4y = y^2x$

yes

$xyxy = \cancel{xy\,xy\,yx^4} = \cancel{xy^4}$

$(xy)^2 \quad yx^4xy = y^2$

$(xy)^3 = xyy^2 = xy^3$

$(xy)^4 = xyxy^3 = xx^4yy^3 = y^4 = e$

~~$(xy)^5 = xy(xy)^4 = xy^5 = xy$~~

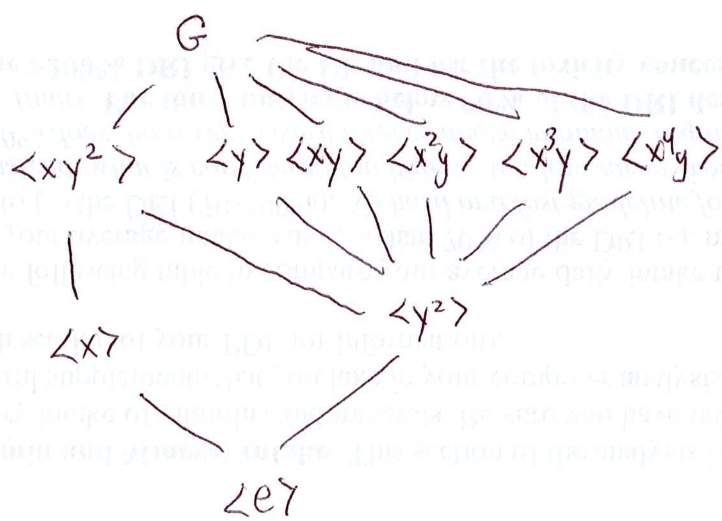
$x^2yx^2y = x^2x^3yy = y^2$

$x^3yx^3y = x^3x^2yy = y^2$

$x^4yx^4y = x^4xyy = y^2$

possible orders 2, 4, 5, 10

a subgp of order 10 must have an element of order so must contain x, x^2, x^3, x^4



~~$y^4 xy^4 x^2y$~~ x^3y, x^4y

e, x, x^2, x^3, x^4

$G = \{$ ~~y, xy, x^2y~~, x^3y, x^4y

~~y^2, xy^2~~, x^2y^2, x^3y^2, ~~x^4y^2~~