4/16

— Gonna create an assignment that is more philosophical than mathematical.

20pts - Quiz

R (comm. ring w/ identity)

Def $a,b \in R$ $(b \neq 0)$ a divides b, $a|b$ if $\exists r \in R$ $ar = b$

$a \nmid b$ doesn't divid

Proposition If $a | bc$ and a,b are ~~comaximal~~ prime relatively prime.

then $a|c$. (Hint use Bezout's identity.

Pf: relatively prime means $\gcd(a,b) = 1$.

co-maximal means $aR + bR = R$

So 1 can be written as an R-lin. comb of $a + b$

$\exists x, y \in R$ $1 = xa + yb$

(2)

Aside

Ex. in $\mathbb{Z}$ $\quad 16, 3.$ $\quad aR \cap bR = 0$

$$1 = 16\cdot(1) + 3(-5). = \cancel{16(2)}$$

$$16(-2) + 3(11)$$

$(*)$ $1 = xa + yb.$ $\quad a|bc$ $\exists r \in R$ $ra = bc$

mult. eqn $(*)$ by c.

$cyb = (bc)y$

$= (ra)y$

$$c = cxa + cyb = \cancel{cxa + cyb} \cancel{ar}$$

$$= cxa + ray =$$

$$= a(cx + ry) \quad \text{so } a|c.$$

R comm. ring. if $a|bc$ and $aR + bR = R$

then $a|c$.

ideal I, J $\quad$ $I + J$ ideal

$(i+j) + (x+y)$

$= (i+x) + (j+y)$

$r(i+j) = ri + rj$

③ Bezout's identity

$\mathbb{Z}$. $\gcd(a,b)$ ~~may b~~ can be written as a lin. comb a, b.

What about rings s.t. $\forall a,b \neq 0$ $\gcd(a,b)$ exists.

about rings w/ $\forall a,b \neq 0$ $\gcd(a,b)$ exists and ~~it~~ it can be written as a lin. comb. of a, b.

a priori

Bezout rings

$\mathbb{Z}$ not a field

R[x] $a_0 + a_1x + \dots + a_nx^n$ R[x] not a field

R field R[x] x has no inv. in R[x]

TFAE 1) R field

2) R[x] Euclidean domain

3) R[x] PID

4) R[x] Bezout

Jaffard Ohm - Kaplansky

(4) F field ~~then~~ and $f(x) \in F[x]$

$\alpha \in F$ is a root of $f(x)$ iff $(x-\alpha) \mid f(x)$.

pf

$$f(x) = (x-\alpha)g(x) \quad \text{some } g(x) \in F[x]$$

$\Rightarrow$

Div. Alg.

$f(x) = (x-\alpha)g(x) + r(x)$

$r = 0$ or

deg $r(x)$ < deg $x-\alpha$

"

1

$$x^2+3x+2 = (x+1)(x+2)$$

$$(x-3)(x^2+3x+7) \qquad \frac{-3 \pm \sqrt{9-28}}{2}$$