rings commutative w/ identity

fields :

integral domains

- subrings
- ideals - ideal gen. by a set
  - principal ideals aR
  - finitely generated ideals $a_1R+\dots+a_nR$
  - prime, maximal

P.I.D. ; Noetherian; Bézout

Prop R int. domain iff $R[x]$ int. domain

Thm $R[x]$ PID iff R field

Noetherian iff a.c.c. ideals

dcc = artinian

dcc $\Rightarrow$ a.c.c.

$R[x]$ Bezout dom iff R field.

Prop in a PID - every non-zero prime is maximal.
- $r \in R$ irreducible iff prime.

Quotient rings : cosets R/I ; I an ideal.

Isomorphism Theorems

Prop 1) R/I domain iff I prime
2) R/I field iff I maximal

Field Theory : E, F fields

Lin. Alg: vector spaces, dimension, basis $\dim_E V =$

$E \leq F$ makes F into an E-vector space. $[F:E] = \dim_E F$

$f(x) \in E[x]$, $\deg f \geq 1$. Let $f(x)$ be irreducible. $\deg f(x) = n$. can assume monic.

$$E \text{ "}\leq\text{" } \underset{\underset{E[\theta]}{\|}}{E[x]/f(x)E[x]} \leftarrow \text{field.} \qquad \theta = x + f(x)E[x]$$

Thm $[E(\theta):E] = n$

Ex. $f(x) = x^2+1$ irr. over $\mathbb{R}$ $F = \mathbb{R}[x]/f(x)\mathbb{R}[x]$ is a field extension of degree 2

$\theta^2 = ??$

Then Galois Theory :

finite fields