Final Exam – MATH 332

Directions: Make sure to show all necessary work to receive full credit. If you need extra space please use the back of the sheet with appropriate labeling. Buena suerte.

1. [5 pts.] A *vector space* is a set which has an addition and a scalar multiplication by a field $\mathbb{F}$ such that

(i)

(ii)

(iii)

(iv)

(v)

(vi)

(vii

(viii)

2. [10 pts.] State the following definitions. $\mathbf{A}$ is an $n \times n$-matrix.

(i) The vector $\mathbf{x}$ is an *eigenvector* of $\mathbf{A}$ corresponding to the scalar λ if

(ii) The *characteristic polynomial* of $\mathbf{A}$ is

(iii) A function $T : V \to W$ between 2 vector spaces V and W is called a *linear transformation* if

(iv) The matrices $\mathbf{A}$ and $\mathbf{B}$ are called *similar* if

(v) A set of vectors $\{x_1, x_2, \cdots, x_n\}$ is said to be linearly dependent if

(vi) The square matrix is called *upper triangular* if

3. [4 pts.] Let $z = 5 - 3\mathbf{i}$. Find $||z||$ and z^{-1}.

4. [7 pts.] Let $\mathbf{A}$ be an $m \times m$ matrix. Suppose that $\mathbf{A} = \mathbf{PDP^{-1}}$ for some invertible matrix $\mathbf{P}$ and diagonal matrix $\mathbf{D}$. Prove by induction that for any natural number n that $\mathbf{A}^n = \mathbf{PD}^n\mathbf{P^{-1}}$.

5. [8 pts.] Let V be a vector space. Prove that if U_1 and U_2 are subspaces of V then $U_1 \cap U_2$ is also a subspace. (Be sure to define $U_1 \cap U_2$!)

6. [10 pts.] Let $\mathbf{A} = \begin{bmatrix} 3 & -1 & 2 \\ 1 & 3 & 4 \\ 2 & -1 & 1 \end{bmatrix}$ Row reduce the matrix. What are the basic and free variables. Write the nul space of A as a span of vectors. What is the dimension of the nul space. Find the determinant of $\mathbf{A}$.

7. [5 pts.] Determine the value of h such that the matrix is the augmented matrix of a consistent linear system. $\mathbf{A} = \begin{bmatrix} 1 & h & -2 \\ -4 & 2 & 10 \end{bmatrix}$

8. [3 pts.] Let $S = \{\begin{bmatrix} 1 \\ -4 \end{bmatrix}, \begin{bmatrix} 5 \\ -2 \end{bmatrix}, \begin{bmatrix} 4 \\ -4 \end{bmatrix}\}$ be 3 vectors in the plane. Is this set a linearly independent set? Explain.

9. [20 pts.] Consider the following matrix over $\mathbb{R}$.

$$\mathbf{A} = \begin{bmatrix} 4 & 4 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

(a) Find the characteristic polynomial for $\mathbf{A}$. What are the eigenvalues of $\mathbf{A}$ and what are their multiplicities? (Graph it and find the roots... you may need to zoom in.)

(b) To each eigenvalue find the dimension of its corresponding eigenspace by finding a basis for the eigenspace. List a basis for each eigenspace.

(c) Is $\mathbf{A}$ diagonalizable? Explain your answer. If it is find an invertible matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A} = \mathbf{PDP}^{-1}$.

10. [10 pts.] Consider the following matrix over $\mathbb{Z}_3$.

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 2 & 2 \end{bmatrix}$$

Find the characteristic polynomial over $\mathbb{Z}_3$. List the eigenvalues of $\mathbf{A}$ if there are any.

11. [8 pts.] Let

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 2 & 3 & 1 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

Write out $\mathbf{A}^t$. Without doing any calculations determine the determinant and eigenvalues of $\mathbf{A}$. Is $\mathbf{A}$ invertible?

Do this page last. Again do these only when you are finsihed with the other problems.

12. [1 pt. for each] State as many of the equivalent conditions listed in the invertible matrix theorem.

13. [10 pts.] Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be defined by $T(x, y, z) = (2x + 3y - z, x + z)$. Prove that T is a linear transformation and find its standard matrix representation. Since there is collapsing what is its nul space?