Exam 3– MATH 332

Directions: Make sure to show all necessary work to receive full credit. If you need extra space please use the back of the sheet with appropriate labeling. Buena suerte.

1. [15 pts.] State the following definitions. Let $\mathbf{A}$ be an $n \times n$ matrix with entries in the field $\mathbb{F}$ and $\mathbf{x}$ an n-dimensional vector.

(i) The vector $\mathbf{x}$ is an *eigenvector* of $\mathbf{A}$ if

(ii) The scalar $\lambda \in \mathbb{F}$ is called an *eigenvalue* of $\mathbf{A}$ if

(iii) The *characteristic polynomial* of $\mathbf{A}$ is

(iv) The matrices $\mathbf{A}$ and $\mathbf{B}$ are called *similar* if

(v) The matrix $\mathbf{A}$ is called *diagonalizable* if

(vi) Given the complex number $z = a + b\mathbf{i}$ find its length, $||z||$, and z^{-1}:

2. [3 pts.] Suppose $z = a + b\mathbf{i}$ and $w = c + d\mathbf{i}$. What is the imaginary part of zw?

3. [5 pts.] Let $z = 3 + 4\mathbf{i}$ and $w = -1 + 4\mathbf{i}$. Find $||zw||$.

4. [10 pts.] Let $\mathbf{A}$ be an $m \times m$ matrix. Suppose that $\mathbf{A} = \mathbf{PDP^{-1}}$ for some invertible matrix $\mathbf{P}$ and diagonal matrix $\mathbf{D}$. Prove by induction that for any natural number n that $\mathbf{A}^n = \mathbf{PD}^n\mathbf{P^{-1}}$.

5. [10 pts.] Suppose $\mathbf{x}$ is an eigenvector of $\mathbf{A}$ corresponding to eigenvalue $\lambda \in \mathbb{F}$. Prove by induction that $\mathbf{A}^k\mathbf{x} = \lambda^k\mathbf{x}$.

6. [15 pts.] Let $\mathbf{A} = \begin{bmatrix} 1 & 3 & -1 \\ 0 & 1 & 2 \\ 2 & 0 & 3 \end{bmatrix}$ Find the determinant of **A**. Find the classical adjoint (using the cofactor algorithm). Then find $\mathbf{A}^{-1}$.

7. [5 pts.] Suppose that

$$\mathbf{A} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} d & e & f \\ 2a & 2b & 2c \\ g & h & i \end{bmatrix}$$

If $\det \mathbf{A} = 10$, then $\det \mathbf{B} =$

8. [5 pts.] Show that if $\mathbf{A} = \mathbf{QR}$ with **Q** invertible, then **A** is similar to **RQ**.

9. [22 pts.] Consider the following matrix over $\mathbb{R}$.

$$\mathbf{A} = \begin{bmatrix} 4 & 2 & 3 \\ -1 & 1 & -3 \\ 2 & 4 & 9 \end{bmatrix}$$

(a) Find the characteristic polynomial for $\mathbf{A}$. What are the eigenvalues of $\mathbf{A}$ and what are their multiplicities? (Graph it and find the roots...)

(b) To each eigenvalue find the dimension of its corresponding eigenspace by finding a basis for the eigenspace. List a basis for each eigenspace.

(c) Is $\mathbf{A}$ diagonalizable? Explain your answer. If it is find an invertible matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A} = \mathbf{PDP}^{-1}$.

Do these last as I may change to bonus problems. Again do these only when you are finsihed with the other problems.

10. [6 pts.] Consider the following matrix over $\mathbb{Z}_7$.

$$\mathbf{A} = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$$

Find the eigenvalues of **A**.

12. [12 pts.] Consider the following matrix over $\mathbb{C}$.

$$\mathbf{A} = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$$

Find the eigenvalues of **A** and find corresponding eigenvectors..