Exam 6– MATH 332 – Summer 2005

Directions: Make sure to show all necessary work to receive full credit. If you need extra space please use the back of the sheet with appropriate labeling. Good luck.

1. Find the characteristic polynomial (or equation) for the matrix

$$\begin{bmatrix} 3 & -2 & 8 \\ 0 & 5 & -2 \\ 0 & -4 & 3 \end{bmatrix}$$

2. Find the characteristic polynomial and the eignevalues of the matrix

$$\begin{bmatrix} 3 & -1 \\ 1 & -1 \end{bmatrix}$$

3. Given an $n \times n$ matrix, say A, explain when 0 is an eigenvalue of A.

4. Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation.

   a) Define the nul space of T, denoted nul(T).

   b) Prove that nul(T) is a subspace.

5. True or False: If $A\mathbf{x} = \lambda\mathbf{x}$ for some scalar λ, then $\mathbf{x}$ is an eignevector of A.

6. The eignevalues of a matrix are on its main diagonal.

7. For a given matrix A an elementary row operation on A does not change the determinant.

8. Let

$$P = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix} \text{ and } D = \begin{bmatrix} 1 & 0 \\ 0 & -13 \end{bmatrix}.$$

If $A = PDP^{-1}$, then find A^4.

9. Consider the matrix

$$A = \begin{bmatrix} 5 & -2 & 6 & -1 \\ 0 & 3 & h & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Find h so that the dimension of the eigenspace corresponding to $\lambda = 5$ is 2.

10. a) Find the characteristic polynomial for the following matrix.

    b) Find the eigenvalues.

    c) Find a basis for the eigenspace for each eigenvalue.

    d) Diagonalize the matrix, if possible. (Otherwise, explain why you cannot.)

$$A = \begin{bmatrix} 3 & -1 \\ 1 & 5 \end{bmatrix}$$

11. a) Find the characteristic polynomial for the following matrix.

    b) Find the eigenvalues.

    c) Find a basis for the eigenspace for each eigenvalue.

    d) Diagonalize the matrix, if possible. (Otherwise, explain why you cannot.)

    [Hint: one of the eigenvalues of A is $\lambda = 1$.]

$$A = \begin{bmatrix} 7 & 4 & 16 \\ 2 & 5 & 8 \\ -2 & -2 & -5 \end{bmatrix}$$