Exam 4– MATH 332 – Summer 2007

Directions: Make sure to show all necessary work to receive full credit. If you need extra space please use the back of the sheet with appropriate labeling. Good luck.

For problems 1. thru 7. circle either true or false.

1. True or False: $\mathbb{R}$ is a subspace of $\mathbb{R}^3$.

2. True or False: For any two $n \times n$ matrices A and B, $\det(AB) = \det(A)\det(B)$.

3. True or False: For an $n \times n$ square matrix A, $L_A : \mathbb{R}^n \to \mathbb{R}^n$ is one-to-one if and only if it is onto.

4. True or False: A basis for the vector space V is a linearly dependent set of vector which spans V.

5. True or False: Suppose A is an $n \times n$ matrix. A is diagonalizable if A has n distinct eigenvalues.

6. True or False: The eignevalues of a triangular matrix are the entries along its main diagonal.

7. True or False: For a given matrix A an elementary row operation on A does not change the determinant.

8. Given an $n \times n$ matrix, say A, explain when $\lambda = 0$ is an eigenvalue of A.

9. Given an $n \times n$ matrix A, define an eigenvector of A.

10. a) Prove that the following function $L : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation.

$$L((x, y)) = (3x - y, x - y).$$

b) Find the standard matrix representation of L.

$$A_L = \begin{bmatrix} & & \\ & & \end{bmatrix}$$

c) Find the characteristic polynomial and the eignevalues of the matrix A_L. (Hint: Use the quadratic formula.)

char(A)= ____________________ Eigenvalues=____________________

11. Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation.

a) Define the nul space of T, denoted nul(T).

$$nul(T) = \{ \qquad\qquad\qquad\qquad\qquad\qquad \}.$$

b) Prove that nul(T) is a subspace.

12. Play the "game" with the following matrix.

$$A = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix}$$

a) Find the characteristic polynomial of A.

b) Find the eigenvalues of A.

c) Find a basis for each eigenspace of A. Then write down and circle each corresponding dimension.

d) Is A diagonalizable? If not explain why not. If so, construct an invertible matrix Q such that $Q^{-1}AQ$ is a diagonal matrix.

13. Play the "game" with the following matrix.

$$A = \begin{bmatrix} 4 & 0 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

a) Find the characteristic polynomial of A.

b) Find the eigenvalues of A.

c) Find a basis for each eigenspace of A. Then write down and circle each corresponding dimension.

d) Is A diagonalizable? If not explain why not. If so, construct an invertible matrix Q such that $Q^{-1}AQ$ is a diagonal matrix.

14. Consider the matrix

$$A = \begin{bmatrix} 5 & -2 & 6 & -1 \\ 0 & 3 & h & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Find h so that the dimension of the eigenspace corresponding to $\lambda = 5$ is 2.

Bonus

15. a) Find the characteristic polynomial for the following matrix.

b) Find the eigenvalues.

c) Find a basis for the eigenspace for each eigenvalue.

d) Diagonalize the matrix, if possible. (Otherwise, explain why you cannot.)

[Hint: one of the eigenvalues of A is $\lambda = 1$.]

$$A = \begin{bmatrix} 7 & 4 & 16 \\ 2 & 5 & 8 \\ -2 & -2 & -5 \end{bmatrix}$$