Exam 3– MATH 332 – Summer 2007

Directions: Make sure to show all necessary work to receive full credit. If you need extra space please use the back of the sheet with appropriate labeling. Good luck.

For problems 1. thru 6. circle either true or false.

1. True or False: $\mathbb{R}^2$ is a subspace of $\mathbb{R}^3$.

2. True or False: For any two $n \times n$ matrices A and B, $\det(A + B) = \det(A) + \det(B)$.

3. True or False: For an $m \times n$ matrix A, Col A is the set of all vectors in $\mathbb{R}^m$ that can be written as Ax for some $x \in \mathbb{R}^n$.

4. True or False: The columns of an invertible $n \times n$ matrix form a basis for $\mathbb{R}^n$.

5. True or False: The dimension of the vector space $\mathbb{P}_5$ is 4.

6. True or False: Given an $p \times q$ matrix B, we know that $nul(B)$ is a subspace of $\mathbb{R}^q$ and Col$B$ is a subspace of $\mathbb{R}^p$.

7. Let $B = \{v_1, \cdots, v_p\}$ be a set of vectors in the vector space V. State the definition of what it means that the set B is a *basis* for V.

8. Let V and W be two vector spaces and $L : V \to W$ be a linear transformation. 1) Define the *nul space of* L, denoted $nul(L)$. 2), Prove that $nul(L)$ is a subspace of V.

$$1) \quad nul(L) = \{ \qquad\qquad\qquad\qquad\qquad\qquad \}.$$

9. Let

$$A = \begin{bmatrix} 1 & -2 & 1 \\ 3 & x & 1 \\ -2 & 6 & x \end{bmatrix}$$

Compute the determinant of A and write it in the space below. (Your answer should be a quadratic polynomial.)

$\det(A) =$ ____________________________.

For which values of x is the matrix A not invertible?

$x =$ ____________________________.

10. Let $A = \begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$. Find $(AB)^{-1}$.

11. Let $L : \mathbb{R}^2 \to \mathbb{R}^4$ be defined by

$$L((x_1, x_2)) = (2x_2 - 3x_1, x_1 - 4x_2, 0, x_2).$$

Prove that T is a linear transformation. Then find the matrix representation of L.

$$A_L = \begin{bmatrix} \quad & \quad \\ \quad & \quad \\ \quad & \quad \\ \quad & \quad \end{bmatrix}$$