Exam 3– MATH 332 – Summer 2005

Directions: Make sure to show all necessary work to receive full credit. If you need extra space please use the back of the sheet with appropriate labeling. Good luck.

Answer problems 1. thru 3. either true or false (circle your answer).

1. If the set of vectors $\{v_1, \cdots, v_k\}$ over $\mathbb{R}^n$ is linearly independent, then we call the set a basis.

2. If $T : \mathbb{R}^3 \to \mathbb{R}^5$ is a linear transformation, then the matrix representation of T, A_T, is a 3×5 matrix.

3. A linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is completely determined by its effect on the columns of the $n \times n$ identity matrix.

4. Let
$$A = \begin{bmatrix} 1 & -4 & 7 & -5 \\ 0 & 1 & -4 & 3 \\ 2 & -6 & 6 & -4 \end{bmatrix}$$
Explain whether the column vectors of A i) span $\mathbb{R}^3$, ii) are linearly independent.

5. Let $T : \mathbb{R}^2 \to \mathbb{R}^4$ be defined by

$$T((x_1, x_2)) = (2x_2 - 3x_1, x_1 - 4x_2, 0, x_2).$$

Prove that T is a linear transformation.

6. Let $A = \begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$. Find $(AB)^{-1}$.

7. Is the vector $\mathbf{b}=\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$ in the span of the column vectors from Problem 4.

8. Find the (standard) matrix representation, A_T, of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by

$$T((x, y, z)) = (x, -y, z).$$

Geometrically what is this linear transformation doing to 3-space? Are the column vectors of A_T a basis for $\mathbb{R}_3$? Explain your answers.