1. Suppose $A$ is a $3 \times 3$ matrix and its characteristic polynomial is $c_B(x) = x^3 - 2x^2 - 21x - 18$. Factor $c_A(x)$ and list the eigenvalue(s) of $B$. [Hint: find a root by checking, then use long division.]

2. Let $A = \begin{pmatrix} -8 & 1 & -1 \\ -20 & 1 & -4 \\ 5 & -1 & -2 \end{pmatrix}$. Find the characteristic polynomial of $A$, $c_A(x)$, in factored form. (Show work.) [Hint: a root must be a divisor of the constant term.] List the eigenvalue(s) of $A$.

3. How do you find the eigenvalues of a triangular matrix?

4. True or False:
   (a) An eigenvalue of the matrix $A$ is a vector $\vec{v}$ such that $A\vec{v} = \lambda \vec{v}$.

   (b) Let $A$ be a $3 \times 3$ matrix. Then ker $A$ is non-trivial.

   (c) Let $T$ be a linear transformation and $A_T$ is matrix representation; $A_T$ is an $n \times n$ matrix. $T$ is injective (i.e. one-to-one) if and only if ker $T$ is trivial.

   (d) If $A$ is a singular matrix, then $A$ must have an eigenvalue.

   (e) Every $3 \times 3$ matrix over the real numbers has an eigenvalue.
5. Let \( E = \begin{pmatrix} 3 & -1 & -1 \\ 4 & -2 & -4 \\ -2 & 2 & 4 \end{pmatrix} \). The characteristic polynomial of \( C \) is \( c_E(x) = (x - 2)^2(x - 1) \).

(a) What are the eigenvalue(s) of \( E \) and what are their algebraic multiplicities?

(b) Compute \( 2I_3 - E \).

(c) Find a basis for the eigenspace corresponding to the eigenvalue \( \lambda = 2 \).

(d) What is the geometric multiplicity of \( \lambda = 2 \)?

(e) Is \( E \) diagonalizable? Explain.
6. Suppose $A$ is a $3 \times 3$ matrix and $c_A(x)$ factors into the nice form $x(x - 1)^2$. You also know that

\[
\beta_0 = \left\{ \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} \right\} \text{ is a basis for } E_0 \text{ and } \beta_1 = \left\{ \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ -5 \end{pmatrix} \right\} \text{ is a basis for } E_1. \text{ Is } A \text{ diagonalizable?}
\]

Let $P = \begin{pmatrix} 2 & -1 & 2 \\ 3 & -1 & 2 \\ 0 & 3 & -5 \end{pmatrix}$. What is $P^{-1}AP$? [Hint: $E_\lambda$ is the eigenspace corresponding to the eigenvalue $\lambda$.]

7. Let $G = \begin{pmatrix} 1 & -2 & -2 \\ 1 & -2 & -1 \\ -1 & 1 & 0 \end{pmatrix}$. Is $G$ diagonalizable? If so, find $P$ such that $P^{-1}AP$ is a diagonal matrix. Be sure to show all work: i) find characteristic polynomial, ii) find eigenvalues, iii) find basis for each eigenspace, iv) construct $P$, v) find the diagonal matrix $P^{-1}AP$. Lastly, find $P^{-1}$.

8. (Extra Credit) Suppose $A$ is a $3 \times 3$ non-scalar matrix and $c_A(x) = (x - r)^3$ for some real number $r$. Can $A$ be diagonalizable? Explain. [A scalar matrix is one of the form $\alpha I_n$ for a constant $\alpha$ and the identity matrix.]