Name: ____

Exam 2 – MAS 2103H Spring 2020

1. Suppose A is a 3×3 matrix and its characteristic polynomial is $c_B(x) = x^3 - 2x^2 - 21x - 18$. Factor $c_A(x)$ and list the eigenvalue(s) of B. [Hint: find a root by checking, then use long division.]

2. Let $A = \begin{pmatrix} -8 & 1 & -1 \\ -20 & 1 & -4 \\ 5 & -1 & -2 \end{pmatrix}$. Find the characteristic polynomial of A, $c_A(x)$, in factored form. (Show work.) [Hint: a root must be a divisor of the constant term.] List the eigenvalue(s) of A.

3. How do you find the eigenvalues of a triangular matrix?

4. True or False:
   (a) An eigenvalue of the matrix A is a vector $\vec{v}$ such that $A\vec{v} = \lambda\vec{v}$.

   (b) Let A be a 3×3 matrix. Then $\ker A$ is non-trivial.

   (c) Let T be a linear transformation and A_T is matrix representation; A_T is an $n \times n$ matrix. T is injective (i.e. one-to-one) if and only if $\ker T$ is trivial.

   (d) If A is a singular matrix, then A must have an eigenvalue.

   (e) Every 3×3 matrix over the real numbers has an eigenvalue.

5. Let $E = \begin{pmatrix} 3 & -1 & -1 \\ 4 & -2 & -4 \\ -2 & 2 & 4 \end{pmatrix}$. The characteristic polynomial of C is

$$c_E(x) = (x-2)^2(x-1).$$

(a) What are the eigenvalue(s) of E and what are their algebraic multiplicities?

(b) Compute $2I_3 - E$.

(c) Find a basis for the eigenspace corresponding to the eigenvalue $\lambda = 2$.

(d) What is the geometric multiplicity of $\lambda = 2$?

(e) Is E diagonalizable? Explain.

6. Suppose A is a 3×3 matrix and $c_A(x)$ factors into the nice form $x(x-1)^2$. You also know that $\beta_0 = \{\begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}\}$ is a basis for E_0 and $\beta_1 = \{\begin{pmatrix} -1 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ -5 \end{pmatrix}\}$ is a basis for E_1. Is A diagonalizable? Let $P = \begin{pmatrix} 2 & -1 & 2 \\ 3 & -1 & 2 \\ 0 & 3 & -5 \end{pmatrix}$. What is $P^{-1}AP$? [Hint: E_λ is the eigenspace corresponding to the eigenvalue λ.]

7. Let $G = \begin{pmatrix} 1 & -2 & -2 \\ 1 & -2 & -1 \\ -1 & 1 & 0 \end{pmatrix}$. Is G diagonalizable? If so, find P such that $P^{-1}AP$ is a diagonal matrix. Be sure to show all work: i) find characteristic polynomial, ii) find eigenvalues, iii) find basis for each eigenspace, iv) construct P, v) find the diagonal matrix $P^{-1}AP$. Lastly, find P^{-1}.

8. (**Extra Credit**) Suppose A is a 3×3 non-scalar matrix and $c_A(x) = (x-r)^3$ for some real number r. Can A be diagonalizable? Explain. [A scalar matrix is one of the form αI_n for a constant α and the identity matrix.]