Name: ____

Exam 2 – MAS 2103H Spring 2019

1. Suppose B is a 3×3 matrix and its characteristic polynomial is $c_B(x) = x^3 + 4x^2 + x - 6$. Factor $c_B(x)$ and list the eigenvalue(s) of B. [Hint: find a root by checking, then use long division.]

2. Let $A = \begin{pmatrix} -5 & 0 & -1 \\ -4 & -3 & -2 \\ 4 & 0 & -1 \end{pmatrix}$. Find the characteristic polynomial of A, $c_A(x)$, in factored form. [Hint: factor by grouping.] List the eigenvalue(s) of A.

3. Let $Z = \begin{pmatrix} 3 & h \\ 4 & 2 \end{pmatrix}$, a 2×2 matrix over $\mathbb{Z}_5$. Find the characteristic polynomial of Z and determine for which values of $h \in \mathbb{Z}_5$, Z is diagonalizable.

4. Let $E = \begin{pmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{pmatrix}$. The characteristic polynomial of C is

$$c_E(x) = (x-2)^2(x-1).$$

(a) What are the eigenvalue(s) of E and what are their algebraic multiplicities?

(b) Compute $2I_3 - E$.

(c) Find a basis for the eigenspace corresponding to the eigenvalue $\lambda = 2$.

(d) What is the geometric multiplicity of $\lambda = 2$?

(e) Is E diagonalizable? Explain.

5. Suppose A is a 3×3 matrix and $c_A(x)$ factors into the nice form $x(x-1)^2$. You also know that $\beta_1 = \{ \begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix} \}$ is a basis for E_0 and $\beta_2 = \{ \begin{pmatrix} -1 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ -5 \end{pmatrix} \}$ is a basis for E_1. Is A diagonalizable? If so, construct a P so that $P^{-1}AP = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. [Hint: E_λ is the eigenspace corresponding to the eigenvalue λ.]

6. True or False:
   (a) An eigenvalue of the matrix A is a non-zero vector $\vec{v}$ such that $A\vec{v} = \lambda\vec{v}$.

   (b) Let A be a 3×4 matrix. Then $\ker A$ is non-trivial.

   (c) Let A be an $n \times n$ matrix. T_A is injective (i.e. one-to-one) if and only if T_A is surjective (i.e. onto).

   (d) If A is a singular matrix, then A must have an eigenvalue.

   (e) The set $\{A \in M_n(\mathbb{F}) : \det(A) = \pm 1\}$ is closed under matrix multiplication.

7. **Extra Credit** Refer to problem 5. Find P^{-1}.

8. **Extra Credit** Refer to problem 5. Find A.