Name: ______________________________

Exam 2– MATH 3320 – Spring 2010

Directions: Make sure to show all necessary work to receive full credit. If you need extra space please use the back of the sheet with appropriate labeling.

1. State the definition for what it means for the set of vectors $\{\mathbf{v_1}, \mathbf{v_2}, \cdots, \mathbf{v_p}\}$ is *linearly independent.* [Be sure to include the correct quantifiers.]

2. Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a function. State the definition for what it means for T to be a *linear transformation.* [Be sure to include the correct quantifiers.]

3. (a) Suppose $f : X \to Y$ is a function. State the definition for what it means for f to be *one-to-one.*

   (b) Suppose $f : X \to Y$ is a function. State the definition for what it means for f to be *onto.*

4. Suppose $T : \mathbb{R}^m \to \mathbb{R}^n$ is a linear transformation. Prove that T is one-to-one if and only if $T\mathbf{x} = \mathbf{0}$ has a unique solution. [Remember that "if and only if" means you should prove two directions.]

5. Consider the function $S : \mathbb{R}^2 \to \mathbb{R}^3$ defined by

$$S((x, y)) = (2x, x - y, 0).$$

Prove that S is a linear transformation.

6. Consider the linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^3$ defined by

$$T((w, x, y, z)) = (w + y + z, 2w - 3z, 4x - 2y).$$

Find the standard matrix representation of T.

7. Let $A = \begin{bmatrix} 3 & 2 & 3 \\ 2 & 0 & 4 \\ 1 & 1 & 0 \end{bmatrix}$ whose entries belong to $\mathbb{Z}_5$. Determine whether A^{-1} exists (over $\mathbb{Z}^5$). If so, find it.

8. Are the column vectors of the following matrix linearly independent? Explain your answer.

$$A = \begin{bmatrix} 4 & 2 & 3 & 3 \\ -1 & 0 & 4 & 1 \\ 1 & 1 & 0 & -2 \end{bmatrix}$$

9. Are the column vectors of the following matrix linearly independent? Explain your answer.

$$B = \begin{bmatrix} 2 & 2 & 3 \\ -1 & 0 & 4 \\ 1 & 1 & 0 \\ 2 & -1 & 5 \end{bmatrix}$$

10. Using A and B from the previous two problems let T_A and T_B be the linear transformations given by $T_A(\mathbf{x}) = A\mathbf{x}$ and $T_B(\mathbf{y}) = B\mathbf{y}$. Determine whether T_A and T_B are one-to-one and/or onto.

11. This problem is a **BONUS** problem, so do this problem last. It will not count anywhere near as the other problems.

Circle your answer to the following questions.

1. True or False: If A is an $m \times n$ matrix and B is an $m \times k$ matrix then the product BA is well-defined.

2. True or False: If the $m \times n$ matrix A has a pivot position in every row, then the equation $A\mathbf{x} = \mathbf{b}$ has a unique solution for every $\mathbf{b} \in \mathbb{R}^m$.

3. True or False: If A and B are $n \times n$ matrices then $(AB)^T = B^T A^T$.

4. True or False: If A and B are invertible $m \times m$ matrices, then so is AB.

5. True or False: If A is a 3×2 matrix, then T_A cannot be one-to-one linear transformation.

6. True or False: If A is a 3×2 matrix, then T_A cannot be an onto linear transformation.

7. True or False: A mapping $T : \mathbb{R}^n \to \mathbb{R}^m$ is one-to-one if each vector in $\mathbb{R}^n$ maps onto a unique vector in $\mathbb{R}^m$.

8. True or False: The vector equation $\mathbf{0} = c_1\mathbf{v_1} + \cdots + c_k\mathbf{v_k}$ has a unique solution if and only if the set of vectors $\{\mathbf{v_1}, \cdots \mathbf{v_k}\}$ is linearly dependent.