Name: _____

Exam 2– MATH 332 – Summer 2007

Directions: Make sure to show all necessary work to receive full credit. If you need extra space please use the back of the sheet with appropriate labeling. Buena suerte.

Answer problems 1. through 5. either true or false (circle your answer).

1. True or False: When $\mathbf{u}$ and $\mathbf{v}$ are nonzero vectors, Span$\{\mathbf{u},\mathbf{v}\}$ contains the line through $\mathbf{u}$ and the origin.

2. True or False: If the equation $A\mathbf{x}=\mathbf{b}$ is inconsistent, then $\mathbf{b}$ is not in the set spanned by the columns of $\mathbf{A}$.

3. True or False: If a set in $\mathbb{R}^n$ is linearly dependent, then the set contains more than n vectors.

4. True or False: If the linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is one-to-one, then $n \leq m$.

5. True or False: The $n \times n$ matrix A has no free variables if and only the columns of A span $\mathbb{R}^n$.

6. Fill in the answer to the following problem:

   The product of a $m \times n$ matrix with a $n \times k$ matrix results in a _____ matrix.

7. Fill in the answer to the following problem:

   "If A is an $m \times n$ matrix, then the columns of A are linearly independent if and only if the reduced echelon form matrix of A has _____ pivot columns.

8. Explain a way of determining whether a given set of column vectors in $\mathbb{R}^n$, say $\{\mathbf{v}_1, \cdots, \mathbf{v}_k\}$, is linearly independent.

9. Compute the product AB of matrices where $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -2 & 1 & x \\ y & -1 & 2 & 0 \\ 1 & 0 & 1 & z \end{bmatrix}$.

10. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that first reflects points through vertical x_2-axis, and then rotates points $\frac{\pi}{3}$ radians (or 60 degrees). Find the standard matrix representation of T.

$$A_T = \begin{bmatrix} & \\ & \\ & \end{bmatrix}$$

11. Determine whether the following function $T : \mathbb{R}^2 \to \mathbb{R}^3$ is a linear transformation or not. If it is, prove it. If not, explain why not.

$$T((x, y)) = (x + 4, y + x).$$

12. Determine whether the following function $T : \mathbb{R}^3 \to \mathbb{R}^2$ is a linear transformation or not. If it is, prove it. If not, explain why not.

$$T((x, y, z)) = (x + 2y, -z).$$