Name: _____

Exam 1– MATH 3320 – Spring 2010

Directions: Make sure to show all necessary work to receive full credit. If you need extra space please use the back of the sheet with appropriate labeling.

(1) Consider the following system of linear equations.

$$\begin{aligned} 5x - 2y + 4z &= 2 \\ 3y - 5z &= 1 \\ 2x - 4y + 7z &= 3 \end{aligned}$$

Write the system as a matrix equation.

Let A be the **augmented** matrix associated to the system of linear equations.

$$A = \left[\begin{array}{ccc|c} & & & \\ & & & \\ & & & \end{array}\right]$$

Find the row reduced echelon form of A.

$$rref(A) = \left[\begin{array}{ccc|c} & & & \\ & & & \\ & & & \end{array}\right]$$

Is the original system of equations consistent or inconsistent (why or why not)? If it is consistent find the solution(s).

(2) a) Explain how to determine whether the homogeneous equation $A\mathbf{x} = \mathbf{0}$ has a trivial solution or not.

b) Let $A = \begin{bmatrix} 0 & 5 & 1 \\ 4 & -3 & 0 \\ 2 & 4 & 1 \end{bmatrix}$. Does the homogeneous equation $A\mathbf{x} = \mathbf{0}$ have a non-trivial solution? Explain your answer.

(3) Let $\mathbf{v_1} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$, $\mathbf{v_2} = \begin{bmatrix} 5 \\ 1 \\ 1 \end{bmatrix}$ and $\mathbf{y} = \begin{bmatrix} 2 \\ h \\ 0 \end{bmatrix}$. For what value(s) of h is $\mathbf{y}$ in the plane generated by $\mathbf{v_1}$ and $\mathbf{v_2}$.

(4) Determine the general solution of the following augmented matrix over $\mathbb{R}$. Circle the pivot positions and label the basic and free variables (as in x_1, x_2, x_3, x_4).

$$\left[\begin{array}{cccc} 1 & 4 & 2 & 10 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

(5) State the definition for what it means for the vector $\mathbf{b}$ to be a linear combination of the vectors $\mathbf{v_1}, \mathbf{v_2}, \cdots, \mathbf{v_k}$.

(6) Suppose $X = \{\mathbf{v_1}, \mathbf{v_2}, \cdots, \mathbf{v_k}\}$ is a set of vectors in $\mathbb{R}^n$. Explain how to determine whether the vector $\mathbf{b}$ is a linear combination of the vectors in X.

(7) This problem takes place over $\mathbb{Z}_5$. Let

$$B = \begin{bmatrix} 1 & 4 & 2 \\ 2 & 1 & 4 \\ 3 & 0 & 3 \end{bmatrix}$$

Row reduce B. (Show work.)

(8) Circle your answer to the following questions.

1. True or False: A $m \times n$ matrix has m rows.

2. True or False: Two $m \times n$ matrices are row equivalent if they have the same number of free variables.

3. True or False: A consistent system has more than one solution.

4. True or False: The reduced echelon form of a matrix is unique.

5. True or False: A basic variable in a linear system is a variable that corresponds to a pivot column in the coefficient matrix.

6. True or False: If two matrices are row equivalent, then they have the same reduced row echelon form.

7. True or False: If A is an $m \times n$ matrix and the equation $A\mathbf{x} = \mathbf{b}$ has a solution for some $\mathbf{b}$, then the columns of A span $\mathbb{R}^m$.

8. True or False: If A is an $m \times n$ matrix and the equation $A\mathbf{x} = \mathbf{b}$ is consistent for every $\mathbf{b} \in \mathbb{R}^m$, then A has m pivot positions

9. True or False: The equation $A\mathbf{x} = \mathbf{0}$ does not have a non-trivial solution if and only if there are no free variables.

10. True or False: The vector $\mathbf{b}$ is a linear combination of the vectors $\mathbf{a_1}, \cdots, \mathbf{a_p}$ if and only if the augmented matrix $[\mathbf{b}\ \mathbf{a_1}\ \cdots\ \mathbf{a_p}]$ is consistent.