Exam 1– MATH 332

Directions: Make sure to show all necessary work to receive full credit. If you need extra space please use the back of the sheet with appropriate labeling. Buena suerte.

Remark 0.1. Throughtout the test $\mathbf{x}, \mathbf{b}, \mathbf{v}, \mathbf{x_1}, \cdots, \mathbf{x_n}$ will denote column vectors.

1. Define the following:

(a) A linear combination of the vectors $\mathbf{x_1}, \cdots, \mathbf{x_n}$ is:

(b) The subset spanned (or generated) by the vectors $\mathbf{x_1}, \cdots, \mathbf{x_n}$ is:

2. a) Find the inverse of 7 mod 13.

3. Add 4mod 7 to 6mod 7.

4. Let

$$\mathbf{A} = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 1 & -1 \\ 4 & 1 & 3 \end{bmatrix}; \qquad \mathbf{B} = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & 1 \\ 3 & 2 & 1 \end{bmatrix}$$

a) Find **AB** (over $\mathbb{R}$.)

b) Find **AB** (over $\mathbb{Z}_5$.)

5. Put the following coefficient matrix in reduced echelon form. What does it say?

$$\begin{bmatrix} 2 & 2 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 4 & 1 & 3 & 0 \end{bmatrix}$$

6. Determine the value of x so that the matrix is an augmented matrix of a consistent linear system (over $\mathbb{R}$.)

$$\begin{bmatrix} 10 & 5 & 3 \\ 4 & 2 & x \end{bmatrix}$$

7. Over $\mathbb{Z}_5$ determine whether $\mathbf{b} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$ is a linear combination of

$$\mathbf{a_1} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}; \mathbf{a_2} = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}; \mathbf{a_3} = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$$

8. Suppose $\mathbf{A}$ is a 2x3 matrix. Does $\mathbf{Ax} = \mathbf{0}$ have a non-trivial solution (why or why not?)

9. $\mathbf{A}$ is a 3x2 matrix and has 2 pivot positions.

a) Does $\mathbf{Ax} = \mathbf{0}$ have a non-trivial solution?

b) Does $\mathbf{Ax} = \mathbf{b}$ have a solution for every $\mathbf{b}$. By the way, what are the dimensions of $\mathbf{b}$, i.e. for what n is $\mathbf{b} \in \mathbb{F}^n$?

10. Solve the following system of equations over $\mathbb{Z}_7$:

$$\begin{aligned} 2x + 3y + 2z &= 2 \\ 4x \qquad\quad - 3z &= 1 \\ 2x + 4y + z &= 3 \end{aligned}$$