Exam 1– MAS 2103H – Spring 2020

Directions: Make sure to show all necessary work to receive full credit. If you need extra space please use the extra blank sheet with appropriate labeling.

1. Compute AB and A^t where $A = \begin{bmatrix} 3 & -3 & -2 & 0 \\ 2 & 5 & 4 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 3 & 1 \\ -2 & 0 & 2 \\ 0 & -2 & 2 \\ 3 & 3 & 1 \end{bmatrix}$.

2. Definitions
   (a) Suppose V and W are $\mathbb{R}$-vector spaces. State what it means for the function $T : V \to W$ to be a linear transformation.

   (b) State the definition of Span$(\vec{v_1}, \vec{v_2}, \ldots, \vec{v_k})$.

   (c) Suppose $T : V \to W$ is a linear transformation between two $\mathbb{R}$-vector spaces. Define the kernel (or nul space) of T.

3. Consider the following system of equations. In the space provided please write down the augmented matrix that would be used to solve the system by using Gaussian. Do not solve the system.

$$\begin{aligned} 3x + 5y + z &= 5 \\ 7x - 2y + 4z &= 1 \\ -6x + 3y + 2z &= 2 \end{aligned}$$

4. Consider the following 3×4 matrix over $\mathbb{R}$. Row reduce the matrix over $\mathbb{R}$.

$$C = \begin{bmatrix} -2 & 2 & 4 & 1 \\ 1 & -1 & 3 & 2 \\ 4 & 0 & 1 & -1 \end{bmatrix}$$

5. Find the determinant of the following matrx

$$C = \begin{bmatrix} 1 & -2 & 2 \\ 3 & -3 & 1 \\ 2 & 2 & -1 \end{bmatrix}$$

6. Consider the matrix over $\mathbb{R}$ and its row reduced echelon form.

$$A = \begin{bmatrix} 1 & 2 & 1 & 4 & 3 \\ -1 & 0 & -3 & 2 & 2 \\ 1 & 2 & 1 & 4 & 0 \\ 2 & -1 & 7 & -7 & 3 \end{bmatrix}; \quad \operatorname{rref}(A) = \begin{bmatrix} 1 & 0 & 3 & -2 & 0 \\ 0 & 1 & -1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(a) Circle the pivot positions in $\operatorname{rref}(A)$. How many free variables are there? What are they?

(b) Find a basis for the kernel of A. Hint: Use the parametric form first.

(c) Do the column vectors of A span all of $\mathbb{R}^4$? Explain.

7. Let $L : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation that does $L((1,0)) = (3,5)$ and $L((0,1)) = (2,-1)$.

(a) What is $L((3,5))$?

(b) Find the matrix representation of L.