Exam 1– MAS 2103 – Spring 2013

Directions: Make sure to show all necessary work to receive full credit. If you need extra space please use the extra blank sheet with appropriate labeling.

(1) True or False: $\mathbb{Z}$ is a field.

(2) True or False: $\mathbb{Z}_5$ is a commutative ring with identity.

(3) True or False. The homogeneous equation $\mathbf{A}\vec{x} = \vec{0}$ always has a solution.

(4) True or False: A $m \times n$ matrix has n columns.

(5) Suppose V and W are $\mathbb{F}$-vector spaces. State what it means for the function $T : V \to W$ to be a linear transformation.

(6) State the definition of $\text{Span}(\vec{v_1}, \vec{v_2}, \ldots, \vec{v_k})$.

(7) Suppose $T : V \to W$ is a linear transformation between two $\mathbb{F}$-vector spaces. Define the kernel of T.

(8) Let $A = \begin{bmatrix} 1 & 2 & 1 \\ 4 & -3 & 0 \\ 1 & -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -2 & 2 \\ 1 & 1 & 2 \\ 3 & -3 & 0 \end{bmatrix}$.

Consider these as matrices over $\mathbb{R}$. Compute A^t and AB.

(9) Consider the following system of linear equations.

$$\begin{aligned} 2x_1 + 4x_2 + 2x_3 &= 2 \\ 3x_1 + \qquad 3x_3 &= 1 \\ 2x_1 + x_2 + 4x_3 &= 3 \end{aligned}$$

Write the system as a matrix equation. Then reduce the matrix to echelon form (or reduced) and find the solution set over $\mathbb{Z}_5$.

(10) Determine whether the function $S : \mathbb{R}^3 \to \mathbb{R}$ is a linear transformation. If it is prove it, find the matrix representation of S, and find the kernel of S. If S is not a linear transformation explain why not.

$$S((x, y, z)) = |x| + y - 2z.$$

(11) You start with a coefficient matrix A and then compute the reduced echelon form given below. Circle the pivot positions and label the free variables and non-free variables (as in x_1, x_2, x_3, x_4).

$$\begin{bmatrix} 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Write the solution set lof the homogeneous equation as a span of a finite number of vectors: (Hint: start with $(x_1, x_2, x_3, x_4)^t$.)

(12) Determine whether the function $T : \mathbb{R}^3 \to \mathbb{R}^4$ is a linear transformation. If it is prove it, find the matrix representation of T, and find the kernel of T. If T is not a linear transformation explain why not.

$$T((x, y, z) = (3x - 2y + z, y - z, y, x + z).$$