Name: ______________________________

Final Exam – MATH 3130 Fall 2009

Directions. Read all directions. Make sure to show any necessary work to receive full credit. Remember to use the back of a sheet if you need to, but label the problem accordingly. If you remove the staple for some reason please put your name on every page.

1. Let $\mathcal{L}$ be a FOL and let P and Q be sentences in $\mathcal{L}$. Answer the following questions regarding P and Q.

   (a) What does it mean when we say P is a literal?

   (b) What does it mean when we say P and Q are logically equivalent?

   (c) What does it mean when we say P is a tautological consequence of Q?

2. Briefly explain the difference between a predicate and a function symbol.

3. State one of the distributive laws.

4. Consider the following values for sentences in the language of Tarski's World:

   1: P is a tautology.
   2: P is a logical necessity.
   3: P is a logical possibility.
   4: P is TT-possibility.
   5: P is not a truth table possibility.

   For each of the following TW-sentences determine the least value for which the sentence makes the value true. Circle each of your responses.

   (a) (SameRow(a, b) $\wedge$ SameCol(b, a)) $\rightarrow a = b$.

   (b) $a = a$.

   (c) (SameSize(a, b) $\wedge$ SameShape(a, b)) $\rightarrow a = b$.

   (d) Larger(a, b) $\vee$ Smaller(a, b) $\vee$ $a = b$.

   (e) $\neg$ (Large(a) $\wedge$ Large(b) $\wedge$ Adjoins(a, b)).

   (f) Large(a) $\wedge$ Large(b) $\wedge$ Adjoins(a, b).

   (g) Medium(a) $\vee$ $\neg$ Medium(b) $\vee$ Medium(b).

   (h) (Small(a) $\rightarrow$ Cube(a)) $\rightarrow$ [(Small(a) $\vee$ Medium(b)) $\rightarrow$ Cube(a)]

5. State what means for an argument to be valid.

6. Supply a Fitch proof for the following Fitch argument. You may not use **Ana Con**. You may use **Taut Con** but only for an instance of the Law of Excluded Middle.

1. $\neg\exists x\forall y\mathbf{P}(x,y)$

2.

3.

4.

5.

6.

7.

8.

9.

10.

11.

12.

13.

14.

15.

16.

17.

18.

19.

20.

21.

22.

23.

24.

25.

26. $\forall x\exists y\neg\mathbf{P}(x,y)$

7. Supply a Fitch proof for the following Fitch argument without premises. You may not use **Ana Con**. You may use **Taut Con** but only for an instance of the Law of Excluded Middle.

1.

2.

3.

4.

5.

6.

7.

8.

9.

11.

12.

13.

14.

15.

16.

17.

18.

19.

20.

21.

22.

23.

24.

25.

26. $(\neg P \vee Q) \leftrightarrow (\neg Q \rightarrow \neg P)$.

8. Supply a Fitch proof for the following Fitch argument without premises. You may not use **Ana Con**. You may use **Taut Con** but only for an instance of the Law of Excluded Middle.

1. _______________
2.
3.
4.
5.
6.
7.
8.
9.
10.
11.
12.
13.
14.
15.
16.
17.
18.
19.
20.
21.
22.
23.
24.
25.
26. $A \vee (B \wedge C) \leftrightarrow (A \vee B) \wedge (A \vee C)$.

9. The following argument is valid. Supply a Fitch proof. You may not use **Ana Con** nor **Taut Con**.

   1. $\exists x \exists y \mathbf{P}(x, y)$

   2. $\forall x \forall y (\mathbf{P}(x, y) \rightarrow \mathbf{Q}(y, x))$

   3.

   4.

   5.

   6.

   7.

   8. $\exists x \exists y \mathbf{Q}(x, y)$

10. Determine whether the following argument is valid. If it is valid supply a Fitch proof. Otherwise supply a counterexample. You may not use **Ana Con** but feel free to use **Taut Con** if it helps.

    1. $\forall y$[Cube(y) $\vee$ Dodec(y)]

    2. $\forall x$[Cube(x) $\rightarrow$ Large(x)]

    3. $\exists x \neg$ Large(x)

    4.

    5.

    6.

    7.

    8.

    9.

    10.

    11.

    12.

    13.

    14.

    15.

    16.

    17. $\exists x$ Dodec(x)

11. Supply a Fitch proof for the following Fitch argument without premises. You may not use **Ana Con**. You may use **Taut Con** but only for an instance of the Law of Excluded Middle.

1. $\forall x\ (\text{Dodec}(x) \rightarrow \text{LeftOf}(x, a))$

2. $\forall x\ (\text{Tet}(x) \rightarrow \text{RightOf}(x, a))$

3. $\forall x \forall y\ (\text{LeftOf}(x, y) \rightarrow \neg\ \text{SameCol}(x, y))$

4. $\forall x \forall y\ (\text{RightOf}(x, y) \rightarrow \neg\ \text{SameCol}(x, y))$

5. $\forall x\ (\text{Cube}(x) \vee \text{Dodec}(x) \vee \text{Tet}(x))$

6.

7.

8.

9.

10.

11.

12.

13.

14.

15.

16.

17.

18.

19.

20.

21.

22.

23.

24.

25.

26.

27.

28. $\forall x\ (\text{SameCol}(x, a) \rightarrow \text{Cube}(x))$

12. Determine whether the following argument is valid. If it is valid supply a Fitch proof. Otherwise supply a counterexample. You may not use **Ana Con** but feel free to use **Taut Con** if it helps.

1. $\forall x$ (Cube(x) $\vee$ Small(x))

2.

3.

4.

5.

6.

7.

8.

9.

10.

11.

12.

13.$\forall x$ Cube(x) $\vee$ $\forall x$ Small(x)

13. Determine whether the following argument is valid. If it is valid supply a Fitch proof. Otherwise supply a counterexample. You may not use **Ana Con** but feel free to use **Taut Con** if it helps.

1. $\forall x \forall y$ [LeftOf$(x, y) \rightarrow$ Larger(x, y)]

2. $\forall x$ [Cube$(x) \rightarrow$ Small(x)]

3. $\forall x$ [Tet$(x) \rightarrow$ Large(x)]

4. $\forall x \forall y$ [(Small$(x) \wedge$ Small$(y)) \rightarrow \neg$ Larger(x, y)]

5.

6.

7.

8.

9.

10.

11.

12.

13.

14.

15.

16.

17.

18.

19.

20.

21.

22.

23.

24.

25. $\forall z \forall w$ [(Tet$(z) \wedge$ Cube$(w)) \rightarrow$ LeftOf(z, w)]

14. Bonus!! Use Fitch to give a formal proof. Do not use **Ana Con**; you may use **Taut Con**.

1. $\exists x\ (\text{Cube}(x) \wedge \forall y\ (\text{Cube}(y) \rightarrow y = x))$

2.

3.

4.

5.

6.

7.

8.

9.

10.

11.

12.

13.

14.

15.

17.

18.

19.

20.

21.

22.

23.

24.

25. $\exists x \forall y\ (\text{Cube}(y) \leftrightarrow y = x)$