Name: _____

Exam 3– MAD 2104H

Directions: Make sure to show any necessary work to receive full credit. If you need extra space please use the extra sheet with appropriate labeling. **TW** means the FOL of Tarski's World

1) Supply a Fitch proof for the following Fitch argument of DeMorgan's Laws. You may not use any of **Ana Con**, **Taut Con**, nor **FO Con**.

1. $\exists x \neg \mathbf{P}(x)$

2.

3.

4.

5.

6.

7.

8.

9.

10. $\neg \forall x \, \mathbf{P}(x)$

2) Supply a Fitch proof for the following Fitch argument of DeMorgan's Laws. You may not use any of **Ana Con**, **Taut Con**, nor **FO Con**.

1. $\forall x \neg \mathbf{P}(x)$

2.

3.

4.

5.

6.

7.

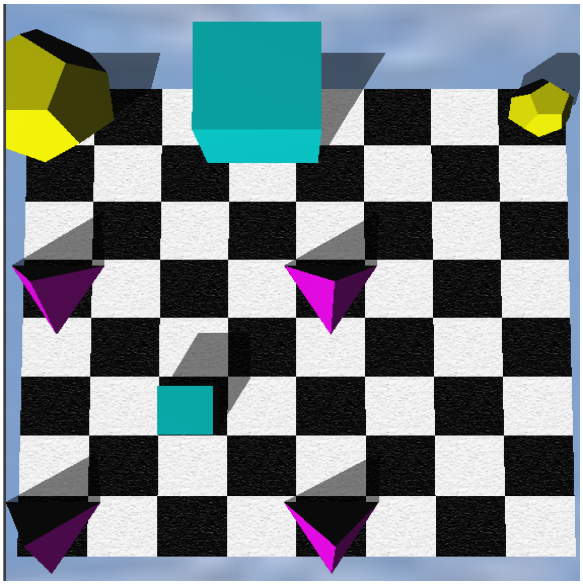
8.

9.

10. $\neg \exists x \, \mathbf{P}(x)$

3) Build a single world where all of the following sentences are true.
   (a) $\exists x$ (Tet(x) $\wedge$ Large(x))
   (b) $\exists x$ (Tet(x) $\wedge$ Medium(x))
   (c) $\exists x$ (Cube(x) $\wedge \neg$Small(x))
   (d) $\exists y$ (Dodec(y) $\wedge \neg$Large(y))
   (e) $\forall x$ (Cube(x) $\rightarrow$ Medium(x))
   (f) $\forall x$ (Dodec(x) $\rightarrow$ Small(x))
   (g) $\forall x$ (Tet(x) $\rightarrow \neg$Small(x))

4) The goal is to try to figure out which objects have names, and what they are. You should be able to figure this out from the sentences, all of which are true.
   (a) Dodec(e)
   (b) $\neg\exists x$ LeftOf(x, e) $\vee$ Large(e)
   (c) FrontOf(d, b) $\wedge$ LeftOf(d, f)
   (d) Between(c, a, d) $\wedge \neg$ Tet(c)
   (e) $\exists x$ BackOf(x, a) $\wedge \exists x$ FrontOf(x, c)
   (f) FrontOf(a, c) $\wedge \neg\exists x$ (BackOf(x, a) $\wedge$ FrontOf(x, c))
   (g) $\exists x$ LeftOf(x, d) $\leftrightarrow$ Large(b)
   (h) $\exists x \exists y$ ($\neg$Tet(x) $\wedge \neg$Tet(y) $\wedge$ Between(b, x, y))



5) Determine whether the following argument is valid or not. If it is valid supply a Fitch Proof. If it is not valid supply a counterexample. Do not use **Taut Con**, **Ana Con**, nor **FO Con**.

1. $\forall y$ [Cube(y) $\vee$ Dodec(y)]
2. $\forall x$ [Cube(x) $\rightarrow$ Large(x)]
3. $\exists x$ $\neg$Large(x)

   $\exists x$ Dodec(x)

6) Determine whether the following argument is valid or not. If it is valid supply a Fitch Proof. If it is not valid supply a counterexample. Do not use **Taut Con**, **Ana Con**, nor **FO Con**.

1. $\forall x$ [Cube(x) $\vee$ (Tet(x) $\wedge$ Small(x))]
2. $\exists x$ [Large(x) $\wedge$ BackOf(x,c)]

   $\forall x$ [Small(x) $\rightarrow$ $\neg$ BackOf(x,c)]

7) Determine whether the following argument is valid or not. If it is valid supply a Fitch Proof. If it is not valid supply a counterexample. Do not use **Ana Con** nor **FO Con**. You may use **Taut Con** but do not apply to wffs, only to complex sentences.

1. $\forall x\ [\mathrm{P}(x) \rightarrow \mathrm{Q}(x)]$
2. $\underline{\forall x\ [\neg\ \mathrm{R}(x) \rightarrow \neg\ \mathrm{Q}(x)]}$

   $\forall x\ [\ (\mathrm{P}(x) \vee \mathrm{Q}(x)) \rightarrow \mathrm{R}(x)\ ]$

8) In this problem we are dealing with the FOL of Set Theory. Suppose you are given the following axioms.

(a) **Extensionality**

$$\forall x \forall y (\forall z (z \in x \leftrightarrow z \in y) \leftrightarrow x = y).$$

(b) **Pairs**

$$\forall x \forall y \exists z \forall v [v \in z \leftrightarrow (v = x \vee v = y)].$$

Notice that the axiom of pairs asserts that for any two sets a, b there is a set whose elements are precisely a and b... that is, substituting a in for x and b in for y produces $z = \{a, b\}$. Also substituting in a for both x and y produces the set $\{a, a\} = \{a\}$. Then substituting in the set $\{a\}$ for x and the set $\{a, b\}$ in for y produces the set $\{\{a\}, \{a, b\}\}$:

So, for any two sets a, b we denote this set by

$$(a, b) = \{\{a\}, \{a, b\}\}.$$

Give an informal argument as to why $(a, b) = (c, d)$ if and only if $a = c$ and $b = d$ so that we can conclude that we can now form ordered pairs. The proof of the sufficiency is easy; which Fitch Rule do you use? For the proof of the necessity consider two cases: i) $a = b$, and ii) $a \neq b$.

9) Translate the following sentences using Tarski's World Predicates and arguments as well as quantifiers.

(a) All cubes are small.

(b) Each cube is either in front of b or in back of a.

(c) No dodecahedron is small.

(d) Nothing is in front of b.

(e) Every cube is also in a different row from every other cube.

(f) There is exactly one large tetrahedron.

(g) Every dodecahedron is smaller than some tetrahedron.

(h) Nothing is between two objects which are the same shape.