Name: _____

Exam 1– MATH 313

Directions: Make sure to show any necessary work to receive full credit. If you need extra space please use the back of the sheet with appropriate labeling.

1. Let $\mathcal{L}$ be a FOL. Define what it means for two sentences, say R and S, to be $\mathcal{L}$-equivalent.

2. Let $\mathcal{L}$ be a FOL. Define what it means when given two sentences, say T and S, that S is an $\mathcal{L}$-consequence of T.

3. Consider the statement: If the two $\mathcal{L}$-sentences P and Q are $\mathcal{L}$-equivalent, then they are tautologically equivalent. Determine whether this statement is true or false. If true, briefly explain why. If false provide a counterexample, i.e. two TW-sentences which are TW-equivalent but not tautologically equivalent.

4. State the definition of a *literal*. Then give an example of a sentence in the FOL of **Set** (i.e., Set Theory) that is not a literal.

5. State the Distributive Laws. (Full credit for both statements.)

   i) **ii)**

6. State De Morgan's Laws. (Full credit for both statements.)

   i) **ii)**

7. Construct a truth table in the space below to prove that one of the De Morgan's Laws that you wrote in 6. is correct. (Your choice).

8. How many predicates are there in **Set** (the FOL of Set Theory)? What are they? Give an example of a sentence in **Set** which is a tautology.

9. Working in **Set** we let $T = \{1, 2, a, b\}$, $S = \{a, b, T\}$, $a = 3$, $b = \{2, 3\}$, $c = \{2, b\}$. Which of the following are true? (In the underlined space provided check the ones that are true.)

   a) $a \in T$ ___ b) $c \in T$ ___

   c) $S \in T$ ___ d) $b \in S$ ___

   e) $c \in c$ ___

10. Consider the following values:

    1: P is a tautology.
    2: P is a TW-necessity.
    3: P is a TW-possibility.
    4: P is truth table possibility.
    5: P is not a truth table possibility.

    For each of the following TW-sentences determine the least value for which the sentence makes the value true. Circle each of your responses.

    (a) $\text{Small}(a) \vee \text{Large}(a) \vee \text{Med}(a)$.

    (b) $\neg\ (\text{Large}(a) \wedge \text{Large}(b) \wedge \text{Adjoins}(a, b))$.

    (c) $a = b \wedge b = b$.

    (d) $\text{SameSize}(a, b) \vee \text{SameShape}(a, b) \vee a = b$.

    (e) $b = b$

    (f) $\text{Medium}(a) \vee \neg\ \text{Medium}(b) \vee \text{Medium}(b)$.

11. Supply a Fitch proof in the space provided. You may not use **Ana Con** nor **Taut Con**.

    1. $\underline{\neg\ (\text{Cube}(b) \vee \text{Tet}(b))}$
    2.
    3.
    4.
    5.
    6.
    7.
    8.
    9.
    10.
    11.
    12.
    13. $\neg\ \text{Cube}(b) \wedge \neg\ \text{Tet}(b)$

12. The following problem presents a formal argument. Determine if the argument is valid. If the argument is valid, supply a Fitch proof in the space provided. If the argument is not valid, supply a counterexample world using Tarski's World. You may not use **Ana Con** nor **Taut Con**.

1. $a = b \wedge b = c \wedge c = d$

2.

3.

4.

5.

6.

7.

8.

9. $a = c \wedge b = d$

Bonus!!! (Only do this after you have finished the other ones.) Write the following sentence in disjunctive normal form.

$$(\text{Small}(a) \vee \text{Medium}(a)) \wedge (\text{Cube}(a) \vee \neg\text{Dodec}(a))$$