Name: ______________________________

Final Exam – MAD 2104H – Fall 2018

Directions: Make sure to show any necessary work to receive full credit. If you need extra space please use the extra sheet with appropriate labeling. **TW**means the FOL of Tarski's World.

1. For each of the following an example of a sentence in Tarski's World:
   (a) A tautology.

   (b) A logical necessity that is not a tautology.

   (c) A logical possibility that is not a logical necessity.

   (d) A TT-possibility that is not a logical possibility.

   (e) A sentence that is not TT-possible.

   (f) An atomic sentence.

   (g) A literal that is not an atomic sentence.

   (h) A well-formed formula (wff).

2. Complete the following truth table.

P	Q	$P \wedge Q$	$P \vee Q$	$P \rightarrow Q$	$P \leftrightarrow Q$	$\neg Q \rightarrow \neg P$
T	T					
T	F					
F	T					
F	F					

3. Given a fixed FOL, state the definition for what it means for two sentences, say P and Q, to be tautologically equivalent. Give an example of two sentences that are tautologically equivalent in the truth table above.

4. Given a fixed FOL, state the definition for what it means for the sentence S to be a logical consequence of the sentence T.

5. Let $S = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}\}$. Is S a natural number? If so, then which one? Construct the power set. Also mention how many elements are in the power set.

$$\mathcal{P}(S) = \{ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \}$$

6. Define an atomic sentence.

7. Given an example in **TW** of a predicate of arrity 2 that is reflexive but not transitive.

8. Given an example in **TW** of a function symbol. State its arrity.

9. What is the difference between a predicate and a function symbol?

10. Supply a Fitch proof for the following Fitch argument. You may not use **Ana Con** nor **FO Con**. You may use **Taut Con** but only for an instance of the Law of Excluded Middle.

$$\frac{[\forall x \mathbf{P}(x)] \to [\forall y \mathbf{Q}(y)]}{[\exists y \neg \mathbf{Q}(y)] \to \neg[\forall x \mathbf{P}(x)]}$$

11. Supply a Fitch proof for the following Fitch argument. You may not use **Ana Con** nor **FO Con**. You may use **Taut Con** but only for an instance of the Law of Excluded Middle.

$\underline{\neg\exists x \forall y \mathbf{P}(x, y)}$

$\forall x \exists y \neg \mathbf{P}(x, y)$

12. Supply a Fitch proof for the following Fitch argument. You may not use **Ana Con** nor **FO Con**. You may use **Taut Con** but only for an instance of the Law of Excluded Middle.

$$\underline{\exists x \forall y \neg \mathbf{P}(x, y)}$$

$$\neg \forall x \exists y \mathbf{P}(x, y)$$

13. Supply a Fitch proof for the following Fitch argument. You may not use **Ana Con** nor **FO Con**. You may use **Taut Con** but only for an instance of the Law of Excluded Middle.

$$\forall x\Big(\mathbf{P}(x) \land \big(\mathbf{Q}(x) \lor \mathbf{R}(x)\big)\Big)$$

$$\forall x\Big[\big(\mathbf{P}(x) \land \mathbf{Q}(x)\big) \lor \big(\mathbf{P}(x) \land \mathbf{R}(x)\big)\Big]$$

14. Supply a Fitch proof for the following Fitch argument. You may not use **Ana Con** nor **FO Con**. You may use **Taut Con** but only for an instance of the Law of Excluded Middle.

$$\exists x\Big(\mathbf{P}(x) \lor \big(\mathbf{Q}(x) \land \mathbf{R}(x)\big)\Big)$$

$$\exists x\Big[\big(\mathbf{P}(x) \lor \mathbf{Q}(x)\big) \land \big(\mathbf{P}(x) \lor \mathbf{R}(x)\big)\Big]$$

15. Translate the following sentences using Tarski's World Predicates and arguments as well as quantifiers.

    (a) All tetrahedrons are not large.

    (b) There is a medium object if front of every other object.

    (c) Every cube has a tetrahedron that is to its right but is neither in front of nor in back of it.

    (d) There is exactly one small cube.

16. Determine whether the following argument is valid or not. If it is valid supply a Fitch Proof. If it is not valid supply a counterexample. Do not use **Taut Con**, **Ana Con**, nor **FO Con**.

    1. $\forall x$ [Cube(x) $\rightarrow$ Small(x)]
    2. $\underline{\forall x \text{ [Adjoins(}x\text{,b)} \rightarrow \text{Small(}x\text{)]}}$

    $\forall x$ [(Cube(x) $\vee$ Small(x)) $\rightarrow$ Adjoins(x,b)]

17. Determine whether the following argument is valid or not. If it is valid supply a Fitch Proof. If it is not valid supply a counterexample. Use **Taut Con** whenever it is convenient but do not use FO Con. You may use **Ana Con**.

    1. $\forall x\ [\text{Small}(x) \rightarrow \text{Cube}(x)]$
    2. $\exists x\ [\text{Tet}(x) \lor \neg\ \text{Tet}(x)]$
    3. $\exists x \neg\ \text{Cube}(x) \rightarrow \exists x\ \text{Small}(x)$

    $\exists x\ \text{Cube}(x)$

18. Use Fitch to give a proof of the following argument without premises. You may use **Taut Con** freely in this proof.

    1. ______________

    $\neg\exists x \forall y \big[E(x, y) \leftrightarrow \neg E(y, y)\big]$