Name: _____

Exam 2 – MAD 2104H – Fall 2019

Directions: Make sure to show any necessary work to receive full credit. If you need extra space please use the extra sheet with appropriate labeling. **TW**means the FOL of Tarski's World.

1. Supply a Fitch proof for the following Fitch argument. You may not use **Ana Con** nor **FO Con**. You may use **Taut Con** but only for an instance of the Law of Excluded Middle.

   $[\forall x \mathbf{P}(x)] \rightarrow [\forall y \mathbf{Q}(y)]$

   $[\exists y \neg \mathbf{Q}(y)] \rightarrow \neg[\forall x \mathbf{P}(x)]$

2. Supply a Fitch proof for the following Fitch argument. You may not use **Ana Con** nor **FO Con**. You may use **Taut Con** but only for an instance of the Law of Excluded Middle.

   $\neg \exists x \forall y \mathbf{P}(x, y)$

   $\forall x \exists y \neg \mathbf{P}(x, y)$

3. Supply a Fitch proof for the following Fitch argument. You may not use **Ana Con** nor **FO Con**. You may use **Taut Con** but only for an instance of the Law of Excluded Middle.

   $\exists x \forall y \neg \mathbf{P}(x, y)$

   $\neg \forall x \exists y \mathbf{P}(x, y)$

4. Supply a Fitch proof for the following Fitch argument. You may not use **Ana Con** nor **FO Con**. You may use **Taut Con** but only for an instance of the Law of Excluded Middle.

   $\forall x \Big(\mathbf{P}(x) \wedge \big(\mathbf{Q}(x) \vee \mathbf{R}(x) \big) \Big)$

   $\forall x \Big[\big(\mathbf{P}(x) \wedge \mathbf{Q}(x) \big) \vee \big(\mathbf{P}(x) \wedge \mathbf{R}(x) \big) \Big]$

5. Supply a Fitch proof for the following Fitch argument. You may not use **Ana Con** nor **FO Con**. You may use **Taut Con** but only for an instance of the Law of Excluded Middle.

   $\exists x \Big(\mathbf{P}(x) \vee \big(\mathbf{Q}(x) \wedge \mathbf{R}(x) \big) \Big)$

   $\exists x \Big[\big(\mathbf{P}(x) \vee \mathbf{Q}(x) \big) \wedge \big(\mathbf{P}(x) \vee \mathbf{R}(x) \big) \Big]$

6. Translate the following sentences using Tarski's World Predicates and arguments as well as quantifiers.

   (a) All tetrahedrons are not large.

   (b) There is a medium object if front of every other object.

   (c) Every cube has a tetrahedron that is to its right but is neither in front of nor in back of it.

   (d) There is exactly one small cube.

   (e) Nothing is in front of b.

7. Determine whether the following argument is valid or not. If it is valid supply a Fitch Proof. If it is not valid, supply a counterexample. Do not use **Taut Con**, **Ana Con**, nor **FO Con**.

   1. $\forall x$ [Cube(x) $\rightarrow$ Small(x)]
   2. $\forall x$ [Adjoins(x,b) $\rightarrow$ Small(x)]

   $\forall x$ [(Cube(x) $\vee$ Small(x)) $\rightarrow$ Adjoins(x,b)]

8. Supply a Fitch proof for the following Fitch argument. You may not use **Ana Con**. You may use (but don't need it) **Taut Con** but only for an instance of a Law of Excluded Middle.

   $\neg\exists x$ **P**(x)

   $\forall x\neg$ **P**(x)

9. Determine whether the following argument is valid or not. If it is valid supply a Fitch Proof. If it is not valid supply a counterexample. Use **Taut Con** whenever it is convenient but do not use FO Con. You may use **Ana Con**.

    1. $\forall x\ [\text{Small}(x) \rightarrow \text{Cube}(x)]$
    2. $\exists x\ [\text{Tet}(x) \vee \neg\ \text{Tet}(x)]$
    3. $\underline{\exists x \neg\ \text{Cube}(x) \rightarrow \exists x\ \text{Small}(x)}$

    $\exists x\ \text{Cube}(x)$

10. Build a single world where all of the following sentences are true.
    (a) $\exists x\ (\text{Tet}(x) \wedge \text{Large}(x))$
    (b) $\exists x\ (\text{Tet}(x) \wedge \text{Medium}(x))$
    (c) $\exists x\ (\text{Cube}(x) \wedge \neg\text{Small}(x))$
    (d) $\exists y\ (\text{Dodec}(y) \wedge \neg\text{Large}(y))$
    (e) $\forall x\ (\text{Cube}(x) \rightarrow \text{Medium}(x))$
    (f) $\forall x\ (\text{Dodec}(x) \rightarrow \text{Small}(x))$
    (g) $\forall x\ (\text{Tet}(x) \rightarrow \neg\text{Small}(x))$

11. **(Extra Credit)** Use Fitch to give a proof of the following argument without premises. You may use **Taut Con** freely in this proof.
    1. $\underline{\qquad\qquad\qquad\qquad}$

    $\neg\exists x \forall y \big[E(x,y) \leftrightarrow \neg E(y,y)\big]$