Name: _____

Exam # 1 – Math 2104H – Fall 2010

Directions: make sure to show work or explain how you got an answer. If using a graph from your calculator explain what steps you took to get the graph. Good luck, buena suerte, bonne chance, geluk, glück.

Part I. Definitions: State the following definitions.

1. State the definition of an *inductive set*.

2. State the definition of a *relation* from A to B.

3. State what it means for the integer a to *divide* the integer b.

4. State what it means for f to be a *function* from A to B ($f : A \to B$).

5. State the definition of an *equivalence relation* on A.

6. State the definition of the *natural numbers*.

7. What is the converse of the statement $P \to Q$?

Part II. Problems:

1. Given the number $n = 1237694x$ find the units digit x which makes this number divisible by 11.

2. Consider the following relation R on the set of natural numbers $\mathbb{N}$ (not the integers).

$$R = \{(n, m) \in \mathbb{N}^2 : n \mid m\}.$$

In other words n is related to m iff n divides m. Prove that R is a partial order on $\mathbb{N}$. Is R a linear order on $\mathbb{N}$? Prove or disprove.

Part III. Proofs: Prove the following statements.

1. Consider the functions $f : A \to B$ and $g : B \to C$. Set $h = g \circ f$. Prove that if h is surjective, then g is surjective.

2. Prove the following by mathematical induction.[Hint: factor out a $(n+1)^2$.]

$$\Sigma_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}.$$

3. Supply a fitch proof for the following argument. Don't forget to support your statements by citing the appropriate lines.

1. | $\forall x$ (Cube(x)$\vee$ Tet(x))

2.| $\exists x$ $\neg$Cube(x)

3.|

4.|

5.|

6.|

7.|

8.|

9.|

10.|

11.|

12.|

13.|

14.| $\exists x$ $\neg$Tet(x)