Name: _____

Exam # 2 – Math 2104H – Fall 2012

Directions: make sure to show work or explain how you got an answer.

1. Given the conditional $P \to Q$ fill in the blanks to the following:

   Inverse :

   Converse:

   Contrapositive:

   What can be said about a conditional statement and its contrapositive?

2. Evaluate the following argument's validity. If it is valid, construct a Fitch proof to show this. If you need to use **Ana Con**, use it only to derive $\perp$ from atomic sentences. You may use Taut Con but only cite at most two sentences at a time. If the argument is invalid, you should construct a counterexample world in the space provided.

   1. $\mathbf{Small}(a) \wedge (\mathbf{Medium}(b) \vee \mathbf{Large}(c))$
   2. $\mathbf{Medium}(b) \to \mathbf{FrontOf}(a, b)$
   3. $\underline{\mathbf{Large}(c) \to \mathbf{Tet}(c)}$
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   16. $\neg\mathbf{Tet}(c) \to \mathbf{FrontOf}(c, b)$

3. Supply a Fitch proof for the following Fitch argument. You may not use **Ana Con**. You may use **Taut Con** but only for an instance of the Law of Excluded Middle (though you don't need it).

   1. $\underline{\neg\exists x\mathbf{P}(x)}$
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   10. $\forall x\neg\mathbf{P}(x)$

4. Supply a Fitch proof for the following Fitch argument. You may not use **Ana Con**. You may use **Taut Con** but only for an instance of the Law of Excluded Middle (though you don't need it).

   1. $\underline{\exists x\neg\mathbf{P}(x)}$
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   12.
   13. $\neg\forall x\mathbf{P}(x)$

5. Supply a Fitch proof for the following Fitch argument without premises. You may not use **Ana Con**. You may use **Taut Con** but only for establishing the Law of Excluded Middle. [If you need more room use the right side of the paper to create a second column.]

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26. $(\neg Q \to \neg P) \leftrightarrow (\neg P \vee Q)$

6. For the following argument, decide whether or not it is valid. If it is, use Fitch to give a formal proof. If it isn't, use Tarski's World to give a counterexample. In this problem you are free to use Taut Con to justify proof steps involving only propositional connectives.

1. $\forall x \forall y((\mathbf{Cube}(x) \wedge \mathbf{Dodec}(y)) \rightarrow \mathbf{FrontOf}(x, y))$

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26. $\forall x(\mathbf{Cube}(x) \rightarrow (\forall y(\mathbf{Dodec}(y) \rightarrow \mathbf{FrontOf}(x, y))))$

7. Supply a Fitch proof for the following argument. You may use **Ana Con** but only where what you are citing and concluding are literals. You may use **Taut Con**.

   1. $\forall x(\mathbf{Cube}(x) \rightarrow (\forall y(\mathbf{Dodec}(y) \rightarrow \mathbf{Larger}(x, y)))$
   2. $\forall x(\mathbf{Dodec}(x) \rightarrow (\forall y(\mathbf{Tet}(y) \rightarrow \mathbf{Larger}(x, y)))$
   3. $\underline{\exists x \mathbf{Dodec}(x)}$
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   25.
   26. $\forall x(\mathbf{Cube}(x) \rightarrow (\forall y(\mathbf{Tet}(y) \rightarrow \mathbf{Larger}(x, y)))$

Bonus!!! Do this page last.

8. Determine whether the following argument is valid or not. If it is, supply a Fitch proof. You are free to use **Taut Con** to justify proof steps involving only propositional connectives. If it isn't valid, supply a counterexample.

   1. $\forall x \forall y((\mathbf{Cube}(x) \wedge \mathbf{Dodec}(y)) \rightarrow \mathbf{Larger}(x, y))$

   2. $\forall x \forall y((\mathbf{Dodec}(x) \wedge \mathbf{Tet}(y)) \rightarrow \mathbf{Larger}(x, y))$

   3.

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   25. $\forall x \forall y((\mathbf{Cube}(x) \wedge \mathbf{Tet}(y)) \rightarrow \mathbf{Larger}(x, y))$