Name: ____

Exam 1– MAD 2104H

Directions: Make sure to show any necessary work to receive full credit. If you need extra space please use the extra sheet with appropriate labeling. **TW** means the FOL of Tarski's World

1] Consider the following FOL sentences. In each, identify: arguments, predicates, function symbols. Determine the arity of the predicate and say whether it is an infix or prefix. Translate the last sentence in the space provided.

(a) Cube(a)

(b) $c = d$

(c) Between(lm(a),a, b)

2] In Fitch, which rule would you use to be able to state that RightOf(a, b) is a consequence of LeftOf(b, a)?

3] Supply a Fitch proof for the following argument. Don't forget to support your statements by citing the appropriate lines. You may use **Ana Con** for reasons of symmetry or transitivity.

1. RightOf(b, c)
2. LeftOf(d, e)
3. <u>$b = d$</u>
4.
5.
6.
7. LeftOf(c, e)

4] What is an *atomic sentence*?

5] State the definition of a *literal*.

6] Given an example in **TW** of a literal that is not an atomic sentence.

7] In **Set**, give an example of a sentence that is not a literal.

8] Complete the following truth table.

P	Q	$P \wedge Q$	$P \vee Q$	$P \rightarrow Q$	$P \leftrightarrow Q$
T	T				
T	F				
F	T				
F	F				

9] Use the space below to create a world where all of the following statements are true.

(a) ¬Tet(f)
(b) ¬SameCol(c, a)
(c) ¬¬SameCol(c, b)
(d) ¬Dodec(f)
(e) c $\neq$ b
(f) ¬(d $\neq$ e)
(g) ¬SameShape(f, c)
(h) SameShape(d, c)
(i) ¬Cube(e)
(j) ¬Tet(c)

10] State the Law of Excluded Middle.

11] State DeMorgan's Laws. (Full credit for both statements.)

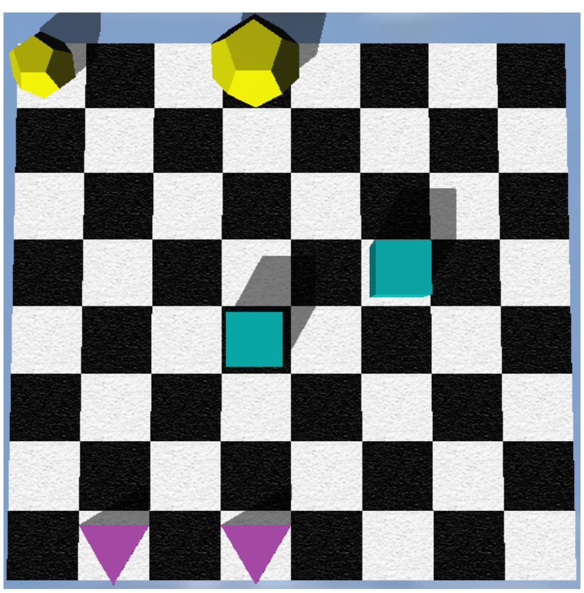
i) **ii)**

12] State the Distributive Laws. (Full credit for both statements.)

i) **ii)**

13] Your task is to assign the names a, b, and c in such a way that all the sentences in the list come out true.

(a) Tet(b) ↔ Tet(c)
(b) Dodec(b) ↔ Dodec(c)
(c) Cube(b) ↔ Cube(c)
(d) Tet(a) ∧ ¬Tet(b)
(e) FrontOf(a, b) → (FrontOf(b, c) ∨ FrontOf(c, b)
(f) LeftOf(a, c) → ¬LeftOf(a, b)
(g) BackOf(b, a) ↔ BackOf(c, b)



14] Consider the following values for sentences in the language of Tarski's World:

I: P is a tautology.
II: P is a logical necessity.
III: P is a logical possibility.
IV: P is TT-possibility.
V: P is not a truth table possibility.

(a) Give an example of a sentence satisfying I.

(b) Give an example of a sentence satisfying II. and not I.

(c) Give an example of a sentence satisfying III. and not II.

(d) Give an example of a sentence satisfying IV. and not III.

(e) Give an example of a sentence satisfying V. and not VI.

(f) Is it possible to give an example of a **TW** sentence that does not satisfy any of the values?

15] Given a fixed FOL, state the definition for what it means for two sentences, say P and Q, to be logically equivalent.

16] Given a fixed FOL, state the definition for what it means for the sentence S to be a tautological consequence of the sentence T (a.k.a. TT-consequence).

17] Consider the statement: "If P is a TT-consequence of Q, then P is a logical consequence of Q." Is this a true statement (yes or no)? If no, then give an example of two **TW** sentences, say P and Q, such that P is a TT-consequence of Q but not a logical consequence of Q.

18] Consider the statement: "If P is a logical consequence of Q, then P is a TT-consequence of Q." Is this a true statement (yes or no)? If no, then give an example of two **TW** sentences, say P and Q, such that P is a logical consequence of Q but not a TT-consequence of Q.

19] For each of the following arguments decide whether the argument is valid. If it is, given an informal proof as to why. If it is not valid, supply a counterexample.

a)

1. Small(a) $\lor$ Small(b)
2. Small(b) $\lor$ Small(c)
3. Small(c) $\lor$ Small(d)
4. Small(d) $\lor$ Small(e)
5. $\neg$Small(c)

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6. Small(a) $\lor$ $\neg$Small(e)

b)

1. Tet(a) $\lor$ $\neg$(Tet(b) $\land$ Tet(c))
2. $\neg$($\neg$Tet(b) $\lor$ $\neg$Tet(d))
3. (Tet(e) $\land$ Tet(c)) $\lor$ (Tet(c) $\land$ Tet(d))

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4. Tet(a)

c)

1. FrontOf(a,b) $\land$ Tet(a)
2. Tet(a) $\to$ Cube(b)
3. $\neg$Cube(b) $\lor$ $\neg$BackOf(b,a)

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4. Cube(a)