Name: _____

Exam # 1 – Math 2104H – Fall 2012

Directions: make sure to show work or explain how you got an answer.

1. Use the space below to create a world where all of the following statements are true.

   (a) ¬ **Tet**(f)
   (b) ¬ **SameCol**(c,a)
   (c) ¬¬ **SameCol**(c,b)
   (d) ¬ **Dodec**(f)
   (e) c≠ b
   (f) ¬ (d≠ e)
   (g) ¬ **SameShape**(f,c)
   (h) ¬¬ **SameShape**(d,c)
   (i) ¬ **Cube**(e)
   (j) ¬ **Tet**(c)

2. State the Distributive Laws.

3. State DeMorgan's Laws.

4. State the definition for what it means for two sentences, say P and Q, to be tautologically equivalent (a.k.a. TT-equivalent).

5. Given a language $\mathcal{L}$ state the definition for what it means for the sentence R to be an $\mathcal{L}$-consequence of the sentence S.

6. State the definitions:

   (a) atomic sentence:

   (b) arrity of a predicate:

   (c) literal:

   (d) function symbol:

   (e) boolean connective:

7. Consider the following values for sentences in the language of Tarski's World:

   1: P is a tautology.

   2: P is a TW-necessity.

   3: P is a TW-possibility.

   4: P is TT-possibility.

   5: P is not a TT-possibility.

   For each of the following TW-sentences determine the least value for which the sentence makes the value true. Circle each of your responses.

   (a) (**SameSize**(a,b) $\wedge$ **SameShape**(b,a)) $\rightarrow$ a=b.

   (b) **Cube**(a) $\vee$ $\neg$**Cube**(b).

   (c) $\neg$(**Larger**(a,b) $\wedge$ **Larger**(b,a)).

   (d) (**Tet**(a) $\rightarrow$ **Large**(a)) $\leftrightarrow$ ($\neg$**Tet**(a) $\vee$ **Large**(a)).

   (e) a=a $\wedge$ b=b.

   (f) $\neg$(**Small**(a) $\wedge$ **Small**(b)) $\vee$ **Small**(a).

   (g) $\neg$(**Large**(a) $\wedge$ **Medium**(a) $\wedge$ **Small**(a)).

8. Fill out the following truth table.

P	Q	$Q \rightarrow P$	$P \rightarrow Q$	$\neg P \vee Q$	$\neg Q \vee P$	$P \leftrightarrow Q$
T	T					
T	F					
F	T					
F	F					

9. Given an example of a pair of TW-sentences, say P and Q, so that P is a TW-consequence of Q, but Q is not a TW-consequence of P.

10. Use the following information to determine the shape and size of the following letters. Fill in the boxes below.

	a	*b*	*c*
Shape:			
Size:			

(a) If *c* is small and *b* is a dodecahedron, then *b* is neither small nor large.

(b) *a* is a tetrahedron but *c* isn't large.

(c) *a* and *c* are both tetrahedron only if at least one of them is large.

(d) If *a* is a tetrahedron then *b* is a dodecahedron.

(e) *c* is a tetrahedron if *b* is a dodecahedron.

(f) *c* is not medium unless *a* is a cube.

11. Supply a Fitch proof of the following valid argument. Do not use **Ana Con** nor **Taut Con**.

1. **FrontOf**(c,e)

2. f=e

3.

4.

5. **FrontOf**(c,f)

12. Determine whether the conclusion is a tautological consequence of the premises. If it is, provide a Fitch proof using one or more applications of **Taut Con**. Do not cite more than two sentences at a time for any of your applications of **Taut Con**. If the conclusion is not a consequence, supply a counterexample.

    1. **Tet**(a) $\vee$ $\neg$(**Tet**(b) $\wedge$ **Tet**(c))

    2. $\neg$ ($\neg$**Tet**(b) $\vee$ $\neg$**Tet**(d)

    3. (**Tet**(e) $\wedge$ **Tet**(c)) $\vee$ (**Tet**(c) $\wedge$ **Tet**(d))

    4.

    5.

    6.

    7.

    8.

    9.

    10.

    11.

    12.

    13.

    14.

    15.

    16. **Tet**(a)