MAD 2104 Honors Discrete Mathematics

First of all please learn the following translations. I will use these in my emails to you.

(1) \wedge is the conjunction symbol $\wedge$
(2) \vee is the disjunction symbol $\vee$
(3) \neg is the negation symbol $\neg$
(4) \perp is the contradiction symbol $\perp$
(5) \ra is the universal quantifier $\rightarrow$
(6) \lra is the universal quantifier $\leftrightarrow$
(7) \forall is the universal quantifier $\forall$
(8) \exists is the existential quantifier $\exists$

## Chapter 1

**Q:** On exercise 1.7 I left the 9th sentence slot open. When I submitted to GradeGrinder it stars it and says "Your ninth sentence slot is empty". Is this telling me that I should be able to construct a sentence that would get the requested result of exactly 3 TRUEs?

**A:** No it is not. It is simply saying that you left it empty. I believe it should be empty.

## Chapter 2

**=Elim** The first two Fitch Proof rules that you are going to learn are the "identity elimination" and "identity introduction". =Elim is used for *substitution.* Up until Chapter 6, you need to know that when you have a sentence $a = b$, then you may substitute the object $b$ for $a$ in any sentence where $a$ is defined. For these first few chapters you are not allowed to plug in $a$ into a sentence involving $b$.

To be perfectly honest, this is done so that you learn to use =Intro. All of the rules come in pairs (Elim and Intro) and so this is there to be consistent. See Page 55 of the text, for more information.

**2.18** So at this point in the book they are saying that the =Elim only works one way .... if you know a=b then you can substitute a for any instance of b. We know that = should be symmetric but they aren't letting you use this yet. Later in the book they will allow you to use symmetry. (The philosophical point they are trying to make is that even a symbol like = must be interpreted. We know we want = to be reflexive, symmetric, and transitive....but that must be built into the symbol on a language.)

**2.20** To get from LeftOf(x,y) to RightOf(y,x) and similar things like this we need to use analytic consequence as in Ana Con. But don't use Ana Con when a different rule (e.g. =Elim) works.

## Chapter 3

**3.28** (Translating from Polish) Try your hand at translating the following sentences from Polish notation into our dialect. Submit the resulting sentence file.

1. NKpq
2. KNpq
3. NAKpqArs
4. NAKpAqrs
5. NAKApqrs

From Page 81: $\neg$P is equivalent to Np, P$\wedge$Q is equivalent to Kpq, and P$\vee$Q is equivalent to Apq.

3. NAKpqArs

Start with the N this is a negation of one thing, and that is of the A which is the disjunction. A is attached to 2 things: The first is K which is also attached to two things Kpq so that is left item to the ∨. Then the other one is Ars which is R∨S.

¬( (P∧Q) ∨ (R∨ S) )

4. Is different than 3 in that the two things attached to the first K is p and Aqr.

¬( (P∧ (Q∨R) ) ∨ S)

## Chapter 4

When doing some truth tables let me remind you what they say on page 101. "we construct the truth table for:

$$P_1 \land P_2 \land P_3 \land \ldots \land P_n$$

as if it were "punctuated" like this:

$$(\ldots(((P_1 \land P_2) \land P_3) \land \ldots \land P_{n-1}) \land P_n$$

**4.9** This problem is asking you to create some truth tables and determine whether some sentences are TT-possible and/or TW-possible. You only need to submit the truth tables. However, it is good for you to be able to decide whether a sentence is TT-possible or not, as well as whether it is TW-possible.

**4.27** The basic idea in these problems with Taut Con is that if you know that the following two sentences are true

$$P \lor Q; \quad \neg P \lor R,$$

then it is a tautological consequence that $Q \lor R$ is true.

**4.30** So the hint is this argument is valid. You are going to learn about DeMorgan's Laws (Chapter 5) and the Distributive Laws (Chapter 4).

De Morgan's Laws say that $\neg(P \lor Q)$ is tautologically equivalent to $\neg P \land \neg Q$. (Distribute negative and flip disjunction to conjunction. Also, $\neg(P \land Q)$ is tautologically equivalent to $\neg P \lor \neg Q$.

The Distributive Laws say that $P \lor (Q \land R)$ is tautologically equivalent to $(P \lor Q) \land (P \lor R$ and dually the other way. For this problem you can get that Tet(b) is true by a Taut Con from line 2. From line 3 with a Tau Con you can derive Tet(c). Then get Tet(b) ∧ Tet(c) and so with that sentence and line 1 you can get Tet(a).

**4.39** A sentence is in disjunctive normal form (DNF) if it is a disjunction of one or more conjunctions of one or more literals.

On 5. the sentence has the form $(P \lor Q \lor R)\land$ S. If you distribute the ∧ then it should be a final disjunction. The distributive law says that it is tautological equivalent to $(P \land S) \lor (Q \land S) \lor (R \land S)$.

## Chapter 6

**6.3** Your premise is a conjunction so you should be able to separate each part into its own sentence using ∧Elim. Then you need use =Elim to gather the other pieces. Then put then back together with ∧ Intro.

**6.4** Premise 1 is a disjunction...you need to consider two cases with an eye on using ∨Elim. So start a subproof with A∧B.... then you can get B and then C∨B. Need another subproof with C....can get C∨B. Then outside of these two subproofs you can get the conclusion that you have in both subproofs using ∨Elim.

**6.5** If A ∧ (B ∨ C), then (A ∧ B) ∨ (A ∧ C).

So this problem is asking for a proof of one of the directions of the distributive laws.

Look at the first sentence. It is a conditional so you can use ∧ Elim to get two sentences A, B∨C. Then the B∨C means you have to consider 2 cases: B or C...so need subproofs for each of these. In the subproof with B true, then you know A∧B by ∧ Intro. Then you can use the ∨ Intro. Similarly, for the other subproof.... remember that in cases you need all of the subproofs to end with the same sentence.

**6.14** Ok so you know that by Premise 1 you should do this proof by cases: 1. SR(b,f), 2. SR(c,f), and 3. SR(d,f)

Case 1. with Premise 3 leads to a contradiction.
Case 2. with Premise 2 is a contradiction.

Once you get a contradiction you can state anything using ⊥ elimination so at the end of each of these subproofs you can conclude that ¬ Cube(f) is true.

In case 3... have a subsubproof with Cube(f). then can get a contradiction with premise 4. so ¬ Intro to get ¬ Cube(f).
Finally, use ∨ Elim.

**6.27** So the premises are 2 disjunctions:
1. $P \vee Q$
3. $S \vee T$
This should make you think of 4 possible cases: $P \wedge S$, $P \wedge T$, $Q \wedge S$, $Q \wedge T$. To do this in Fitch you would use line 1. to create two cases:
Case 1: $P$
Case 2: $Q$
But then you have two subcases in each of these cases so it should look like
Case 1: $P$ Case 1a: $S$ Case 1b: $T$
Case 2: $Q$ Case 2a: $S$ Case 2b: $T$
At the end of each of the cases you would use ∨Elim and line 2.
Then to get out of case 1 and 2 you would us ∨Elim with line 1.

**6.30** So when you do problems from now on you should know about DeMorgan's Laws (p.83).
Premise 1 is equivalent to Cube(a) ∨¬ Cube(b).
Premise 2 is equivalent to Cube(b) ∧¬ Cube(c).
Of course you can ∧ Elim line 2. to get separately that Cube(b).
Now, if you are not allowed to use Taut Con then you need to get Cube(b) the quick way which is to assume (subproof) that ¬ Cub(b) is true and then get ¬ Cube(b) ∨ Cube(c), and then a contradiction. Therefore, Cube(b).
You should then be able to assume (subproof) ¬ Cube(a) and get a contradiction from Premise 1.

## Chapter 7

**7.12.6.** *If e is a tetrahedron, then it's to the right of b if and only if it is also in front of b.* This a conditional sentence if P, then Q. Here the P=Tet(e), while the Q= RightOf(e,b) ↔ Front(e,b)

**7.12.7.** *If b is a dodecahedron, then if it isn't in front of d then it isn't in back of d either.* Dodec(b) → (¬FrontOf(b,d) → ¬BackOf(b,d))

**7.17** Look at sentence 2. $e \neq f \rightarrow$ (Adjoins(b, c) ∧ Adjoins(e, b) ∧ Adjoins(b, f)).

The simplest way to make a conditional true is to make the hypothesis false... so can e=f? Look at 3. This then tells you that e≠ f and so you know the three adjoining relationships.

Rewrite sentence 4. It has the form

$$\neg((P \wedge Q \wedge R) \rightarrow \neg S)$$

change the conditional (the inside) to a disjunction. Then change the negation of conjunctions to a disjunction so that the inside is a long disjunction. Then negate to get a long conjunction.

**7.25** 2. You want to convert the conditional to a disjunction with negations by using the reverse of DeMorgan's Laws.

For sentence 4. use that a conditional $P \rightarrow Q$ is equivalent (tautologically) to $\neg P \vee Q$.

For 6. use that a biconditional $P \leftrightarrow Q$ is first equivalent to the conjunction $(P \rightarrow Q) \wedge (Q \rightarrow P)$, and second, you can do what you did to 1.

For 8. realize that sentence 7. is a conditional (what you did to 3.) where the antecedent is a conjunction (do what you did to 1.) and the consequent is a biconditional (what you did to 5.).

## Chapter 8

**8.25** Transitivity of the Biconditional: From A ↔ B and B ↔ C, infer A ↔C.

So your proof is not correct for the following reason. To prove that A ↔ C you need to have a subproof that starts with A and ends with C. Then you need another subproof that starts with C and ends with A.

**8.29** (P → Q) ↔ ( ¬ P ∨ Q)

Think of it as a puzzle. The first thing to notice is that it is a biconditional; they want you to prove a biconditional. So you need two subproofs. One starting with (P → Q), and ending with ( ¬P ∨ Q). And then another subproof starting with ( ¬P ∨ Q) and ending with (P → Q).

Now once you get to the first case: start with P → Q. Look at the conclusion of ¬P ∨ Q. This is a disjunction. When proving a disjunction you want to think of a Law of Excluded Middle. You may use Taut Con. So Taut Con ¬P ∨ P, and then consider two cases. So you will need two more sub-sub-proofs (within the subproof). First is ¬P...but then ¬P ∨ Q is easy by ∨ Intro. The second case is P....but then this case will give you Q by the hypothesis of the sub-proof...and then also ¬P ∨ Q.

Next, you need a subproof starting with ¬P ∨ Q and then ending with P→Q. Since the conclusion there is a conditional you a subsubproof starting with P and ending with Q.

**8.30** You are starting with ¬(P→ Q) and want to prove that P∧¬ Q.

So look at the conclusion...it is a conjunction. That means you want to prove P separately and ¬ Q separately. How about starting a subproof with ¬ P and trying to get a contradiction....well the only contradiction would be to contradict ¬(P→ Q)...that means you need to show P→ Q in a subproof...so start a subsubproof with P.... but this contradicts your assumption in the subproof that ¬ P is true...so you get your contradiction, then you can state Q.... then outside the subsubproof you can state P→ Q which then contradicts ¬(P→ Q). Thus P is true.

Can you do ¬ Q by assuming Q in a subproof and then contradicting ¬(P→ Q) by showing with a subsubproof with P and ending with Q (Reit)?

**8.34** As for 8.34, the conclusion is Small(a)∧ Large(b). So you need to prove separately that Small(a) is true and then that Large(b) is true. Then you get conclusion by ∧ Intro.

Now look at premise 1. This gives two cases, and the second case gives you Small(a) The first premise will actually give you a contradiction....you can get separately that Tet(a) and Large(a). Tet(a) ∨ Intro gives Tet(a)∨ Cube(a) which by premise 3, Large(b) ∨ Small(b). But also by 4. Medium(b)... ⊥

So all you need is to get Large(b) in that second case. Use 3. to get Large(b) ∨ Small(b). Then have two cases... first case Large(b)...good. 2nd case contradicts premise 2.

**8.52** So you assume Dodec(b) in line 2. Great...your goal is to get a≠ c so that you use → Intro.

In order to get a≠ c so start a subproof with a=c...with the goal of getting a contradiction so you can use contradiction Elim.

But if a=c, then you should be able to get Cube(a) ↔ Cube(c). (You might need to prove that by having a subproof with Cube(a), then use =Elim to say that Cube(c) and vice-versa, and then use iff Intro.

Once you get Cube(a) ↔ Cube(c).... you can get Cube(b). This is within the assumption that Dodec(b) so Ana Con to get your contradiction.

## Chapter 9

**9.11** To make sentence 1. ∃ x (Dodec(x) ∧ Large(x)) false and sentence 2. ∃ x (Dodec(x) → Large(x)) true you need a world with no large dodecs. The thing is if you actually have a dodec then by 2. it will have to be large. So as long as you have an object that is not a dodec 2. will be true. (Remember that in a conditional if the antecedent is false then the conditional is true.) Furthermore, if sentence 1. is true, then the Large dodec witnessed by 1. will also satisfy sentence 2.

Now if sentence 3. ∀ x (Tet(x) ∧ Small(x)), then everything is a small tet. Therefore, if you have a tet (which everything is) it is automatically small. So 4. must also be true.

You should only be submitting a world for the first part of the problem. It is telling me "A text file must be turned in to complete this exercise correctly. This message was not sent to the student." So I will update the file and remove it from the list to be counted as graded.

**9.17** 10. *No dodecahedron is small.* This says that if you have a dodec, then it cannot be small: ∀ x (Dodec(x) → ¬ Small(x)

**9.25** The square of any prime other than 2 is odd.

You would want to think of this as if $x$ is a prime and it is different than 2, then $x^2$ is odd.

So using quantifiers you can try something like

$$\forall y((\text{Prime}(y) \wedge y \neq (1+1)) \rightarrow \neg\text{Even}(y \times y))$$

## Chapter 13

**13.2** The goal in 13.2 is ∀ x Small(x). To prove this you should start a subproof, click on the triangle and elect a name for your arbitrary object...I will say a.

Now line 2 says everything is a cube. So since in the subproof we have an arbitrary object which for the time being we have called a, then we can say that a is a cube. This is how the ∀ Elim works. All you need to do is cite Premise 2.

In the same way if you use ∀Elim on Premise 1 to say that Cube(a) ↔ Small(a).

Now use together the line with Cube(a) ↔ Small(a) and the line with Cube(a) to say Small(a). You can now leave the subproof. Since you started with an arbitrary object and conluded it was small you can now ∀Intro your goal.

**Q:** I tried to claim that ∃¬Cube(x) was a tautological consequence of ¬∀x Cube(x). Why is this not the case?

**A:** It has to do with to tautologies come from truth tables and aren't really involved with qualifiers. We want you to learn a different way to deal with qualifiers. When it comes to quantifiers we say it is a order consequence and not a tautological consequence.

**13.32**

In the video I used A.C. to go from ¬ Small(a) to Large(a)∨ Medium(a). However, the directions say "give a proof that uses Ana Con but only where the premises and conclusions of the citation are literals (including ⊥)." You also may use Taut Con.

So yes in my video I use A.C. to get from ¬ Small(a) to conclude Large(a)∨ Medium(a). Since the conclusion is not a literal this is the problem. Modify the steps in my video. Line 7 is ¬ Small(a) which is fine. In 8, start a subproof with ¬( Large(a) ∨ Medium(a) ) In 9. use T.C. to get that ¬ Large ∧¬ Medium(a). In 10 and 11 use ∧ Elim to stare that ¬ Large(a) , ¬ Medium(a), separately. Now they are literals and so A.C. on 8, 10, 11 produces a contradiction. Then end subproof and get that Large(a) ∨ Medium(a).

**13.38** The conclusion is ∀x (Cube(x) → ∀y (Tet(y) → Larger(x, y))), so start a subproof with Cube(a) (or whatever object you want. You want to end this subproof with ∀y (Tet(y) → Larger(x, y))). So start a subsubproof with Tet(b) and then in this subsubproof you want to end with Larger(a,b).

Here is a different way I answered it: The conclusion is a conditional with the premise a universally quantified sentence. So start a subproof with an object b and the sentence Cube(b). You want to show the conclusion which is a universally quantified sentence. So then start another subproof with c as a Tet.

Now the aim is to get Larger(b,c).

You have to use Premise 3 to say there is a dodec...call it a.

Then use the other two premises one by one to get that Larger(a,b) and then that Larger(b,c).