Make-Up Exam 4– MAC 2312 – Spring 2013

Directions: For the multiple choice part make sure you clearly label your answer. On the free response part make sure to show all necessary work to receive full credit. If you need extra space please use the extra blank sheet with appropriate labeling.

1. Find the length of the curve defined by $x(t) = \frac{1}{2}t^2$ and $y(t) = \frac{1}{9}(6t+9)^{\frac{3}{2}}$, from $t = 0$ to $t = 2$.

   (a) 8 (b) 10 (c) 12 (d) 14 (e) 16

2. A function $f(x)$ has a Maclaurin series $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + \frac{x^{2n}}{(2n)!} + \cdots$. Which of the following is an expression for $f(x)$.

   (a) $\cos x$ (b) $e^x - \sin x$ (c) $e^x + \sin x$ (d) $\frac{1}{2}\left(e^x + e^{-x}\right)$ (e) $e^{x^2}$

3. For time $t > 0$, the position of a particle moving in the $xy$-plane is given by the parametric equations $x(t) = 4t + t^2$ and $y(t) = \frac{1}{3t+1}$. What is the acceleration vector of the particle at time $t = 1$.

   (a) $(2, \frac{1}{32})$ (b) $(2, \frac{9}{32})$ (a) $(5, \frac{1}{4})$ (a) $(6, \frac{-3}{16})$ (a) $(6, \frac{-1}{16})$

4. The $n$th derivative of a function $f(x)$ at $x = 0$ is given by $f^{(n)}(0) = (-1)^n \frac{n+1}{(n+2)2^n}$ for all $n \geq 0$. Which of the following is a Maclaurin series for $f(x)$?

   (a) $\frac{-1}{2} + \frac{1}{3}x - \frac{3}{32}x^2 + \frac{1}{60}x^3 - \cdots$

   (b) $\frac{1}{2} - \frac{1}{3}x + \frac{3}{16}x^2 - \frac{1}{10}x^3 + \cdots$

   (c) $\frac{1}{2} + \frac{1}{3}x + \frac{3}{32}x^2 + \frac{1}{60}x^3 + \cdots$

   (d) $\frac{1}{2} - \frac{1}{3}x + \frac{3}{32}x^2 - \frac{1}{60}x^3 + \cdots$

   (e) $\frac{1}{2} - 3x + \frac{32}{3}x^2 - 60x^3 + \cdots$

5. A curve $C$ is defined by the parametric equations $x(t) = t^2 - 4t + 1$ and $y(t) = t^3 - 9t$. For which values of $t$ does the curve have a horizontal tangent line?

   (a) $t = -\sqrt{3}, \sqrt{3}$ (b) $t = 2$ (c) $t = 3$ (d) $t = -3, 0, 3$ (e) $t = 0$

6. Let $f(x)$ be the function given by $f(x) = \ln(x-3)$. The third degree Taylor polynomial for $f(x)$ about $x = 2$ is

   (a) $-(x-2) + \dfrac{(x-2)^2}{2} - \dfrac{(x-2)^3}{3}$

   (b) $-(x-2) - \dfrac{(x-2)^2}{2} - \dfrac{(x-2)^3}{3}$

   (c) $(x-2) + (x-2)^2 + (x-2)^3$

   (d) $(x-2) + \dfrac{(x-2)^2}{2} + \dfrac{(x-2)^3}{3}$

   (e) $(x-2) - \dfrac{(x-2)^2}{2} + \dfrac{(x-2)^3}{3}$

7. Given the equation of an ellipse $\dfrac{(x-1)^2}{144} + \dfrac{(y-1)^2}{169} = 1$. Where are the foci of the ellipse?

   (a) $(5,0),(-5,0)$ (b) $(25,0),(-25,0)$ (c) $(0,25),(0,-25)$ (d) $(1,6),(1,-4)$ (e) $(1,26),(1,-24)$.

8. Which of the following best describes the following graph of the equation $3(x-1)^2 = 6 + 2(y+1)^2$

   (a) circle (b) parabola (c) ellipse (d) hyperbola (e) hyperplane

9. The vertex of the parabola $y = 5x^2 - 10x + 5$ occurs at

   (a) $(5,-10)$ (b) $(1,2)$ (c) $(0,5)$ (d) $(1,0)$ (e) $(2,5)$

10. For what values of $t$ does the curve given by the parametric equations $x(t) = t^3 - t^2 - 1$ and $y(t) = t^4 + 2t^2 - 8t$ have a vertical tangent line?

    (a) 0 only (b) 1 only (c) 0 and $\frac{2}{3}$ only (d) 0, $\frac{2}{3}$, and 1 (e) No value

11. **Bonus** The Maclaurin series for $f(x) = \dfrac{2}{2-x}$ is

    (a) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{2^n}$ (b) $\displaystyle\sum_{n=0}^{\infty} (-1)^n x^n$ (c) $\displaystyle\sum_{n=0}^{\infty} 2(1-x)^n$ (a) $\displaystyle\sum_{n=0}^{\infty} (1-x)^n$ (a) $\displaystyle\sum_{n=0}^{\infty} \frac{(1-x)^n}{2}$

12. Find the vertex, focus, and directrix of the given parabola.

$$x^2 + 4x + 4y - 4 = 0.$$

13. A particle is moving along a curve in the $xy$-plane so that its position at time $t$ is $(x(t), y(t))$ where

$$\frac{dx}{dt} = \cos(\pi t) \qquad \frac{dy}{dt} = 2\sin(\pi t)$$

for $0 \leq t \leq 2$. At time $t =$ the object is at the point $(,)$.

(a) Find the acceleration vector of the particle at time $t = \frac{1}{2}$.

(b) Find the speed of the particle at time $t = 1$.

(c) Find the slope of the tangent line to the curve at time $t$. For which values of $t \in [0, 2]$ does the curve have a horizontal tangent line?

(d) At time $t = \frac{1}{4}$ is the particle moving to the left or the right?

(e) Use your calculator and find the total distance traveled by the particle from $t = 0$ to $t = 2$.

14. **Bonus!!**

Let $f(x) = e^{-x^2}$.

(a) Write the first four nonzero terms and the general term of the Maclaurin series for $f(x)$.

(b) Use the series from (a) to write the first four nonzero terms and the general term of the Maclaurin series for $1 - x^2 - f(x)$.

(c) Use the series from (b) to write the first four nonzero terms and the general term of the Maclaurin series for $g(x) = \dfrac{1 - x^2 - f(x)}{x^4}$.

(d) Use the series from (c) to write the first four nonzero terms and the general term of the Maclaurin series for $\int_0^x g(t)dt$.