Make-Up Exam 3– MAC 2312 – Spring 2013

Directions: For the multiple choice part make sure you clearly label your answer. On the free response part make sure to show all necessary work to receive full credit. If you need extra space please use the extra blank sheet with appropriate labeling.

1. For what values of x does the series $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n} x^n$ converge?

   (a) $-1 \leq x \leq 1$ (b) $-1 < x \leq 1$ (c) $-e \leq x < e$ (d) $-1 \leq x < 1$ (e) $-e < x \leq e$

2. The power series $1 + 2x + 4x^2 + 8x^3 + \ldots + 2^n x^n + \ldots$ converges for which values of x?

   (a) $x = 0$ only (b) $-\frac{1}{2} < x < \frac{1}{2}$ (c) $-2 < x < 2$ (d) $-1 < x < 1$ (e) all real numbers

3. $\sum_{k=0}^{\infty} \left(\sin\left(\frac{\pi}{6}\right)\right)^k =$

   (a) 1 (b) $\frac{1}{1 - \frac{\sqrt{3}}{2}}$ (c) $\frac{\frac{\sqrt{3}}{2}}{1 - \frac{\sqrt{3}}{2}}$ (d) 2 (e) diverges

4. Which of the following statements are true?

   I. If $\lim_{n\to\infty} a_n = 0$, then the series $\sum_{n=1}^{\infty} a_n$ converges.

   II. If the series $\sum_{n=1}^{\infty} a_n$ satisfies $\lim_{n\to\infty} \frac{a_n}{a_{n+1}} = \frac{1}{2}$, then the series converges.

   III. A telescoping series converges.

   a) I. only b) I. and II. only c) II. and III. only d) III. only e) I., II., and III.

5. Which of the following series converge?

   I. $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

   II. $\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{2^n + 2}$

   III. $\sum_{n=1}^{\infty} \frac{n^3}{4^n}$

   a) I. only b) I. and III. only c) II. and III. only d) III. only e) I., II., and III.

6. For $-1 < x < 1$ which of the following is an anti-derivative of $f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{2n-1}}{2n-1}$?

(a) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{2n-2}}{2n+1}$ (b) $\sum_{n=1}^{\infty} (-1)^n x^{2n-2}$

(c) $\sum_{n=1}^{\infty} \frac{(-1)^{2n} x^{2n}}{2n(2n-1)}$ (d) $\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n}}{2n}$

(e) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{2n}}{2n(2n-1)}$

7. Given that for $-1 < x < 1$, $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$, then $\frac{-1}{(1-x)^2} =$

(a) $\sum_{n=0}^{\infty} x^{2n}$ (b) $\sum_{n=1}^{\infty} n x^{n-1}$

(c) $\sum_{n=1}^{\infty} n x^n$ (d) $\sum_{n=1}^{\infty} n x^{2n}$

(e) $\sum_{n=1}^{\infty} n x^{2n-1}$

8. What are all the values of x for which the series $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n^2}$ converges?

(a) $-1 \le x < 1$ (b) $-1 \le x \le 1$ (c) $0 < x < 2$ (d) $0 \le x < 2$ (e) $0 \le x \le 2$

9. Which of the following statements are true? Assume that $0 < a_n, b_n$.

I. If $\sum_{n=1}^{\infty} a_n$ converges and $\lim_{n\to\infty} \frac{a_n}{b_n} = 2$, then $\sum_{n=1}^{\infty} b_n$ converges.

II. If $\lim_{n\to\infty} a_n = 0$, then the series $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ converges.

III. $\sum_{n=0}^{\infty} \frac{1}{2^n} = 2$

(a) III. only (b) II. only (c) I. and III. only (d) II. and III. only (e) I., II., and III.

10. Which of the following sequences converge?

I. $\{\frac{1}{\sqrt{n}}\}$ II. $\{\frac{\sin n}{n}\}$ III. $\{n!\}$

(a) I. only (b) II. only (c) I. and II. only (d) I. and III. only (e) I., II., and III.

11. **Bonus** What are all the values of x for which the series $\sum_{n=1}^{\infty} \frac{(-1)^n (x-1)^n}{n \cdot 5^n}$ converges?

(a) $-4 < x \le 6$ (b) $-4 \le x \le 6$ (c) $-5 \le x \le 5$ (d) $-5 < x < 5$ (e) $-4 \le x < 6$

12. Compute the sum of the series

$$\sum_{n=3}^{\infty} \frac{1}{n^2-4}$$

13. Find the radius of convergence for $f(x)$. Then find $f(x)$, $f'(x)$, and $\int f(x)dx$ as well as their intervals of convergence.

$$f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-2)^{2n+1}}{2n+1}$$

14. **Bonus!!** Determine whether the following series diverges or converges. Explain your answer by correctly using an appropriate test.

$$\sum_{n=1}^{\infty} \frac{n!}{1 \cdot 3 \cdot 5 \cdots (2n-1)}$$