Final Exam – MAC 2312 – Spring 2013

Directions: For the multiple choice part make sure you clearly label your answer. On the free response part make sure to show all necessary work to receive full credit. If you need extra space please use the extra blank sheet with appropriate labeling.

1. Convert the point in the plane $(-1,-1)$ in rectangular coordinates to polar coordinates.

   (a) $(\sqrt{2}, \frac{5\pi}{4})$ (b) $(-\sqrt{2}, \frac{\pi}{4})$ (c) $\frac{2\pi}{4}$ (d) $(1, \frac{5\pi}{4})$ (e) 12

2. $\displaystyle\int_0^1 \frac{x^2}{x^2+1} =$

   (a) $\dfrac{4-\pi}{4}$ (b) $\ln 2$ (c) 0 (d) $\frac{1}{2}\ln 2$ (e) $\dfrac{4+\pi}{4}$

3. The region bounded by the $x$-axis and the part of the graph of $y = \cos x$ between $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$ is separated into two regions by the line $x = k$. If the area of the region for $-\frac{\pi}{2} \le x \le k$ is three times the area of the region for $k \le x \le \frac{\pi}{2}$, then $k =$

   (a) $\arcsin(\frac{1}{4})$ (b) $\arcsin(\frac{1}{3})$ (c) $\dfrac{\pi}{6}$ (d) $\dfrac{\pi}{4}$ (e) $\dfrac{\pi}{3}$

4. What is $\displaystyle\lim_{x\to 0} \frac{e^{2x}-1}{\tan x}$?

   (a) -1 (b) 0 (c) 1 (d) 2 (e) limit does not exist

5. What is the limit of the sequence $\{\left(1+\dfrac{1}{2n}\right)^{2n}\}$?

   (a) 1 (b) 0 (c) $e$ (d) $e^2$ (e) limit does not exist

6. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$ is the Maclaurin series for which of the following functions?

   (a) $\sin x$ (b) $\cos x$ (c) $e^x$ (d) $e^{-x}$ (e) $\ln(1+x)$.

7. For which values of $x$ does the serie $1 + 2^x + 3^x + 4^x + \cdots n^x + \cdots$ converge?

   (a) No values of $x$ (b) $x < -1$ (c) $x \ge -1$ (d) $x > -1$ (e) all values of $x$

8. (Use Integration by Parts to answer this question.) If $\int x^2 \cos x dx = f(x) - \int 2x \sin x dx$, then $f(x) =$

   (a) $2\sin x + 2x\cos x + C$

   (b) $x^2 \sin x + C$

   (c) $2x\cos x - x^2 \sin x + C$

   (d) $4\cos x - 2x\sin x + C$

   (e) $(2 - x^2)\cos x - 4\sin x + C$

9. A curve $C$ is parameterized by $x(t) = t$ and $y(t) = \tan t$. Which of the following integrals gives the length of the curve between $t = a$ and $t = b$, where $0 < a < b < \frac{\pi}{2}$

   (a) $\int_a^b \sqrt{t^2 + \tan^2 t} dt$

   (b) $\int_a^b \sqrt{t + \tan t} dt$

   (c) $\int_a^b \sqrt{1 + \sec^2 t} dt$

   (d) $\int_a^b \sqrt{1 + \tan^2 t} dt$

   (e) $\int_a^b \sqrt{1 + \sec^4 t} dt$

10. The interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(x+1)^n}{n^2}$ is

    (a) $0 < x < 2$ (b) $0 \le x \le 2$ (c) $-2 < x \le 0$ (d) $-2 \le x < 0$ (e) $-2 \le x \le 0$

11. A curve $C$ is defined by the parametric equations $x(t) = t^2 - 4t + 1$ and $y(t) = t^3$. Which of the following is an equation of the line tangent to the graph of $C$ at the point $(-3, 8)$? [Hint: You need to find $t$.]

    (a) $x = -3$ (b) $x = 2$ (c) $y = 8$ (d) $y = -\frac{27}{10}(x + 3)$ (e) $y = 12(x + 3) + 8$

12. Which of the following is the Maclaurin series for $f(x) = \dfrac{\sin x}{x}$.

(a) $1 - \dfrac{x^2}{3!} + \dfrac{x^4}{5!} - \dfrac{x^6}{7!} + \cdots$

(b) $\dfrac{1}{x} - \dfrac{x}{2!} + \dfrac{x^3}{4!} - \dfrac{x^5}{6!} + \cdots$

(c) $1 + \dfrac{x^2}{3!} + \dfrac{x^4}{5!} + \dfrac{x^6}{7!} + \cdots$

(d) $\dfrac{1}{x} + \dfrac{x}{2!} + \dfrac{x^3}{4!} + \dfrac{x^5}{6!} + \cdots$

(e) $x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \dfrac{x^7}{7!} + \cdots$

13. A particle moves in the $xy$-plane so that at any time $t$ its coordinates are $x = t^2 - 1$ and $y = t^4 - 2t^2$. At time $t = 1$, its acceleration vector is

(a) $(0, -1)$ (b) $(0, 12)$ (c) $(2, -2)$ (d) $(0, 2)$ (e) $(2, 8)$

14. Which of the following is equal to $\displaystyle\int \frac{1}{\sqrt{25 - x^2}} dx$?

(a) $\arcsin \dfrac{x}{5} + C$ (b) $\arcsin x + C$ (c) $\dfrac{1}{5} \arcsin \dfrac{x}{5} + C$ (d) $\sqrt{25 - x^2} + C$ (e) $2\sqrt{25 - x^2} + C$

15. For $-1 < x < 1$ if $f(x) = \displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{2n-1}}{2n - 1}$, then $f'(x) =$

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} x^{2n-2}$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n} x^{2n-2}$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^{2n} x^{2n}$

(d) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n} x^{2n}$

(e) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} x^{2n}$

16. The base of a solid is the region enclosed by the graph of $y = e^{-x}$, the coordinate axes, and the line $x = 3$. If all plane cross sections perpendicular to the $x$-axis are squares, then its volume is

(a) $\frac{(1-e^{-6})}{2}$ (a) $\frac{1}{2}e^{-6}$ (a) $e^{-6}$ (a) $e^{-3}$ (a) $1 - e^{-3}$

17. Which of the following statements are true? Assume that $0 < a_n, b_n$.

I. If $\sum_{n=1}^{\infty} a_n$ converges and $\lim_{n\to\infty} \frac{a_n}{b_n} = 2$, then $\sum_{n=1}^{\infty} b_n$ converges.

II. If $\lim_{n\to\infty} a_n = 0$, then the series $\sum_{n=1}^{\infty} (-1)^n a_n$ converges.

III. $\sum_{n=0}^{\infty} \frac{1}{2^n} = 2$

(a) II. only (b) III. only (c) II. and III. only (d) I. and III. only (e) I., II., and III.

18. Use that for $0 < x \leq 2$, $f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}(x-1)^n}{n}$ to find $f^{(8)}(1)$.

(a) $\frac{-1}{7}$ (b) $\frac{-1}{8}$ (c) $-7!$ (d) $8!$ (e) $-8!$

19. The curve $C$ is parameterized by the equations $x(t) = t^2 - t + 2$, $y(t) = t^3 - 3t$. Find the equation of the tangent line at the point $(4, 2)$ (when $t = 2$).

(a) $y - 2 = \frac{1}{3}(x - 4)$

(b) $y + 2 = \frac{1}{3}(x - 4)$

(c) $y = 2$

(d) $x = 4$

(e) $y - 2 = 3(x - 4)$

20. The volume of the solid obtained by rotating the graph $y = \sin x$ about the $x$-axis between $x = 0$ and $x = \pi$ is exactly

(a) $\frac{\pi}{2}$ (b) $\frac{\pi^2}{2}$ (c) $\pi$ (d) $\frac{493}{100}$ (e) 1

21. **Bonus!!** $\lim_{h\to 0} \frac{\int_1^{1+h} \sqrt{x^5+8}dx}{h}$ is

(a) 0 (b) 1 (c) 3 (d) $2\sqrt{2}$ (e) does not exist

**Free Response**

22. Let $R_1$ be the region in the first quadrant bounded by the graphs of $y = x^{1/3}$ and $y = x^2$. Let $R_2$ be the region in the first quadrant bounded by graphs of $y = x^2$, $x = 1$, and the $x$-axis.

a) Find the volume of the solid generated by rotating region $R_1$ about the $x$-axis.

b) Find the volume of the solid generated by rotating region $R_2$ about the $y$-axis.

c) **Bonus** Find the volume of the solid generated by rotating region $R_2$ about the line $y = 1$.

d) **Bonus** Find the volume of the solid generated by rotating region $R_2$ about the line $x = 1$.

23. Find the integral

$$\int \frac{x^3 - x + 3}{x^2 + x - 2} dx$$

24. Find the center, foci, and vertices of the ellipse

$$9x^2 + 4y^2 + 36x - 24y + 36 = 0$$

25. Given the parameterized curve $x(t) = \sin t + t^2$, $y(t) = e^t + t^2$.

(a) Find the velocity vector. Find the acceleration vector. Find the speed at time $t = \pi$.

(b) Find $\frac{dy}{dx}$.

26. Let $f(x) = \dfrac{\cos x - 1}{x^2}$.

(a) Write the first three nonzero terms and the general term of the Maclaurin series for $\cos x$.

(b) Use the series from (a) to write the first three nonzero terms and the general term of the Maclaurin series for $f(x)$.

(c) Write the first three nonzero terms and the general term of the Maclaurin series for $f'(x)$.

(d) Does the following series converge? Explain.

$$\sum_{n=0}^{\infty} \frac{\sqrt{n^{3/2} + n}}{n^2 + 1}$$

**Bonus!!** (e) Let $g(x) = 1 + \int_0^x f(t)dt$. Write the Maclaurin series for $g$ about $x = 0$.