Name: ______________________________

Final Exam – MAC 2312H – Fall 2019

Directions: For the multiple choice part make sure you clearly label your answer. On the free response part make sure to show all necessary work to receive full credit.

1. Evaluate $\int_1^{\infty} x^{-5}\ dx$

   (A) 1 (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{\pi^2}{12}$ (E) divergent

2. $\int \frac{1}{x^2-5x-6}\ dx =$

   (A) $\frac{1}{7}\ln|x-6| - \frac{1}{7}\ln|x+1| + C$

   (B) $\ln|x^2-5x-6| + C$

   (C) $\frac{1}{6}\ln|x-6| - \frac{1}{6}\ln|x+1| + C$

   (D) $\ln|x-3| - \ln|x-2| + C$

   (E) $\frac{1}{3x^3} - \frac{1}{10x^2} - \frac{1}{6x} + C$

3. $\int 3x\cos(3x)\ dx =$

   (A) $x\cos(3x) - \frac{1}{3}\sin(3x) + C$

   (B) $\frac{1}{3}x\cos(3x) - 3x\sin(3x) + C$

   (C) $\frac{1}{9}x\sin(3x) + C$

   (D) $x\sin(3x) + \frac{1}{3}\cos(3x) + C$

   (E) $3\sin(3x) + C$

4. Find the sum of the infinite series $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}\left(\frac{\pi}{3}\right)^{2n+1}$

   (A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $-\frac{\sqrt{3}}{2}$ (D) $-\frac{1}{2}$ (E) diverges

5. Find the interval of convergence for the power series $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n2^n}$.

   (A) $(-\infty,\infty)$ (B) $[-2,2)$ (C) $[-1,3)$ (D) $[-1,3]$ (E) $[-2,2]$

6. If $x(t) = t^2-1$ and $y = 2e^t$, then $\frac{dy}{dx} =$

   (A) $\frac{e^t}{t}$ (B) $\frac{2e^t}{t}$ (C) $\frac{t}{e^t}$ (D) $\frac{6e^t}{t^3-t}$ (E) e^t

7. Find the sum of the convergent series $\sum_{n=1}^{\infty} \frac{2}{n^2+8n+15}$.

(A) $\frac{9}{20}$ (B) 1 (C) 2 (D) $\frac{1}{4}$ (E) $\frac{1}{12}$

8. Find the limit:

$$\lim_{n\to\infty} \left(\frac{2n+1}{2n+3}\right)^n$$

(A) $e^{\frac{1}{2}}$ (B) ∞ (C) e^2 (D) e (E) e^{-1}

9. A particle moving in the plan has the position function $f(t) = (2t^2 - 1, 4t)$. What is the speed of the particle at time $t = 1$?

(A) $2\sqrt{2}$ (B) $2\sqrt{5}$ (C) 4 (D) $4\sqrt{2}$ (E) $4\sqrt{5}$

10. Which of the following series converge?

I. $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\ln n}$ II. $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ III. $\sum_{n=1}^{\infty} \frac{2\cdot 5\cdot 8\cdots(3n-1)}{1\cdot 3\cdot 5\cdots(2n-1)}$

(a) I. only (b) I. and II. only (c) III. only (d) II. and III. only (e) I., II., and III.

11. Which of the following is the Maclaurin series of the function $f(x) = \ln(1-2x)$ and its radius of convergence R.

(a) $-\sum_{n=1}^{\infty} \frac{2^{n+1}}{n} x^n;\ R = \frac{1}{2}$

(b) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}2^n}{n} x^n;\ R = 2$

(c) $-\sum_{n=1}^{\infty} \frac{2^n}{n} x^{n+1};\ R = \frac{1}{2}$

(d) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}2^n}{n} x^n;\ R = \frac{1}{2}$

(e) $-\sum_{n=1}^{\infty} 2^{n+1} x^n;\ R = 2$

12. What is the coefficient of x^3 in the Maclaurin series for $g(x) = (1+x)^{\frac{1}{3}}$. [Use the Binomial Theorem.]

(A) $-\frac{10}{27}$ (B) $\frac{10}{27}$ (C) -7 (D) $\frac{1}{6}$ (E) $\frac{5}{81}$

13. Find the acceleration of the parametrized function defined by $x(t) = -4t^2 + 2t - 1$ and $y(t) = 6t$.

(A) (0,6) (B) $(-8t+2, 6)$ (C) (-8,6) (D) (-8,0) (E) (0,-8)

14. What is the arc length of the curve parametrized by the equations

$$x(t) = \ln|\sec t|, \qquad y(t) = t, \qquad 0 \le t \le \frac{\pi}{4}$$

(A) $\ln|1 - \frac{1}{\sqrt{2}}|$ (B) $\ln|\sqrt{5} - 2|$ (C) $\ln|\sqrt{2} + 1|$ (D) $\ln|2 - \sqrt{2}|$ (E) $\ln|\sqrt{3} - 1|$

15. For $-1 \le x \le 1$ if $f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{2n-1}}{2n-1}$, then the Maclaurin series and interval of convergence for $f'(x) =$ is

(a) $\sum_{n=1}^{\infty} (-1)^{n+1} x^{2n-2};\ I = [-1, 1]$

(b) $\sum_{n=1}^{\infty} (-1)^{n+1} x^{2n-2};\ I = (-1, 1)$

(c) $-\sum_{n=1}^{\infty} (-1)^{2n} x^{2n};\ I = [-1, 1]$

(d) $\sum_{n=1}^{\infty} (-1)^{n} x^{2n};\ I = (-1, 1]$

(e) $-\sum_{n=1}^{\infty} (-1)^{n+1} x^{2n};\ I = (-1, 1)$

16. $\int \frac{x^2}{x^2+1}\, dx =$

(A) $\ln|x^2 + 1| + C$

(B) $\frac{1}{2}\ln|x+1| - \frac{1}{2}\ln|x-1| + C$

(C) $x - \arctan x + C$

(D) $\frac{x^3}{3}\arctan(x) + C$

(E) $\ln|x+1| - \ln|x-1| + C$

17. $\int \sin^2 x\, dx =$

(A) $\frac{1}{3}\sin^3 x + C$

(B) $\frac{1}{3}x^3 \sin(2x) + C$

(C) $\frac{1}{3}x^3 \cos(2x) + C$

(D) $\frac{1}{2}x - \frac{1}{4}\sin(2x) + C$

(E) $\frac{1}{2}x - \frac{1}{2}\sin(2x) + C$

18. Determine the limit of the sequence $\{a_n\}$ where $a_n = \frac{2\ln n}{5n+4}$.

    (A) $+\infty$ (B) $\frac{2}{5}$ (C) 0 (D) $\frac{5}{9}$ (E) $\frac{1}{2}$

19. Evaluate the integral $\displaystyle\int_e^{\infty} \frac{1}{x(\ln x)^2}\, dx$

    (A) 1 (B) 2 (C) 3 (D) $\ln 2$ (E) diverges to $+\infty$

20. Which of the following statements are true? Assume that $0 < a_n, b_n$.

    I. If $\displaystyle\sum_{n=1}^{\infty} a_n$ converges and $\displaystyle\lim_{n\to\infty} \frac{a_n}{b_n} = 2$, then $\displaystyle\sum_{n=1}^{\infty} b_n$ converges.

    II. If $\displaystyle\lim_{n\to\infty} a_n = 0$, then the series $\displaystyle\sum_{n=1}^{\infty} a_n$ converges.

    III. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n} = 2$

    (A) I. only (B) III. only (C) II. and III. only (D) I. and III. only (E) I., II., and III.

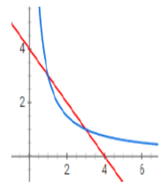
21. $\displaystyle\lim_{x\to 0} \frac{e^{2x} - 1}{\tan x}$

    (A) does not exist (B) -1 (C) 1 (D) 0 (E) 2

22. Evaluate the following indefinite integral. Show work.

$$\int \cos^2 x \sin^3 x\, dx$$

23. Let S be a solid having as base the region in the first quadrant enclosed by the curve $y = \frac{3}{x}$ and the line $y = 4 - x$. Suppose further that parallel cross-sections of S perpendicular to the x-axis are rectangles having as base the vertical line connecting the graphs, and having height twice the base. Find the volume of S. Show work. Find where the graphs intersect.



24. Find the lateral surface area of the volume generated when the curve $y = x^3$ from $x = 0$ to $x = 1$ is rotated around the x-axis. Show work.

25. For what value of k, if any, is $\int_0^\infty kxe^{-2x}\,dx = 1$? Show work.

26. Let $f(x) = x^2$. Find the length of the curve from $x = 0$ to $x = 2$. [Hint: use a trig sub.]

27. Let $f(x) = e^x \sin x$. Compute the second degree Taylor polynomial $p_2(x)$ centered at $x = 0$. [Hint: Compute $f'(x), f''(x)$.]

28. Find the interval of convergence of the Taylor series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (x-1)^{2n}}{4^n n^3}$.

29. Let $f(x) = 2\cosh x = e^x + e^{-x}$.

(a) Write out the first 3 non-zero terms and the general term of the Maclaurin series for e^x.

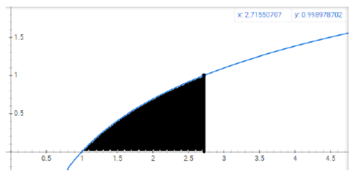
(b) Write out the first 3 non-zero terms and the general term of the Maclaurin series for e^{-x}.

(c) Use (a) and (b) to write the first 3 non-zero terms and then general term for the Maclaurin series for $f(x)$.

(d) Use (c) to write the Maclaurin series for an anti-derivative $F(x)$ of $f(x)$.

(e) What is $\lim_{x \to 0} \dfrac{F(x) - 2x}{x^3}$?

30. Let R be the region in the first quadrant between the graph of $y = \ln x$ and the x-axis between $x = 1$ and $x = e$.



i) **Extra Credit** Use Integration by Parts to get that $\int (\ln x)^2\ dx = x(\ln x)^2 - 2x \ln x + 2x + C$.

ii) **Extra Credit** Use Integration by Parts to get that $\int x \ln x\ dx = \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$.

iii) Find the volume of the solid obtained by rotating R around the x-axis.

iv) Find the volume of the solid obtained by rotating R around the y-axis.

[Hint: we use parts $u = \ln x$, $dv = dx$ to get that $\int \ln x\ dx = x(\ln x - 1) + C$]

Formulas

(1) $\dfrac{1}{1-x} = \sum_{n=0}^{\infty} x^n;\ I = (-1,1)$

(2) $e^x = \sum_{n=0}^{\infty} \dfrac{x^n}{n!};\ I = (-\infty,\infty)$

(3) $\sin x = \sum_{n=0}^{\infty} \dfrac{(-1)^n x^{2n+1}}{(2n+1)!};\ I = (-\infty,\infty)$

(4) $\cos x = \sum_{n=0}^{\infty} \dfrac{(-1)^n x^{2n}}{(2n)!};\ I = (-\infty,\infty)$

(5) $\ln(1+x) = \sum_{n=1}^{\infty} \dfrac{(-1)^{n+1} x^n}{n};\ I = (-1,1]$

(6) $\arctan x = \sum_{n=0}^{\infty} \dfrac{(-1)^n x^{2n+1}}{2n+1};\ I = [-1,1]$

(7) $(1+x)^r = \sum_{n=0}^{\infty} \binom{r}{n} x^n;\ I = (-1,1)$

(8) $\displaystyle\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$

(9) $\displaystyle\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$

(10) $\displaystyle\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{x}{a} + C$

(11) $\dfrac{d}{dx}(\text{sech}^{-1} x) = \dfrac{-1}{x\sqrt{1+x^2}}$

(12) $\displaystyle\int \sec x\ dx = \ln|\sec x + \tan x| + C$

(13) $\displaystyle\int \sec^3 x\ dx = \frac{1}{2}(\ln|\sec x + \tan x| + \tan x \sec x) + C$

(14) $\displaystyle\int \tan x\ dx = \ln|\sec x| + C$

(15) $\displaystyle\int u\ dv = uv - \int v\ du$

(16) $\sin^2 x = \frac{1-\cos(2x)}{2} \quad \cos^2 x = \frac{1+\cos(2x)}{2}$

(17) $\cos^2 x + \sin^2 x = 1 \quad 1 + \tan^2 x = \sec^2 x$

(18) $\sin(x+y) = \sin x \cos y + \cos x \sin y$
(19) $\cos(x+y) = \cos x \cos y - \sin x \sin y$
(20) $\sin(2\theta) = 2\sin\theta\cos\theta$
(21) $\cos(2\theta) = 1 - 2\sin^2\theta$
(22) $\binom{r}{n} = \frac{r(r-1)(r-2)\cdot\ldots\cdot(r-n+1)}{n!}$

(23) $\cosh x = \dfrac{e^x + e^{-x}}{2} \qquad \frac{d}{dx}\cosh x = \sinh x$

(24) $\sinh x = \dfrac{e^x - e^{-x}}{2} \qquad \frac{d}{dx}\sinh x = \cosh x$

Trig Substitutions:

i. Use $x = a\sin\theta$ for expressions with $\sqrt{a^2 - x^2}$
ii. Use $x = a\tan\theta$ for expressions with $\sqrt{a^2 + x^2}$
iii. Use $x = a\sec\theta$ for expressions with $\sqrt{x^2 - a^2}$

Parametrized Curves:

(1) Given $f(t) = (x(t), y(t))$, then velocity is $v(t) = (x'(t), y'(t))$ and acceleration is $a(t) = (x''(t), y''(t))$.

(2) Length of Parametrized Curve: $\int_{t_1}^{t_2} \sqrt{x'(t)^2 + y'(t)^2}\ dt$

(3) Speed / Parametrized Curve at time t: $\sqrt{x'(t)^2 + y'(t)^2}$

(4) $\dfrac{dy}{dx} = \dfrac{y'(t)}{x'(t)}$

Volumes and Surface Areas:

(1) Cross-sections: $\displaystyle\int_a^b A(x)\ dx$

(2) Cylindrical Shells: $\displaystyle\int_a^b 2\pi x f(x)\ dx$

(3) Arc length: $\displaystyle\int_a^b \sqrt{1 + f'(x)^2}\ dx$

(4) Disc/Washer: $\displaystyle\int_a^b \pi\big(f(x)^2 - g(x)^2\big)\ dx$

(5) Lateral Surface Area: $\displaystyle\int_a^b 2\pi f(x)\sqrt{1 + f'(x)^2}\ dx$