Name: _____

Final Exam – MAC 2312 – Fall 2014

Directions: For the multiple choice part make sure you clearly label your answer. On the free response part make sure to show all necessary work to receive full credit. If you need extra space please use the extra blank sheet with appropriate labeling.

1. Consider a solid S whose base is the region enclosed by the x-axis, the curve $y = \sqrt{x}$, and the vertical line $x = 3$. The solid's parallel cross sections perpendicular to the x-axis are squares. The volume of the solid is

   (A) $\frac{9}{2}$ (B) 9 (C) 18 (D) 27 (E) 54

2. Find the volume of the solid generated by rotating about $y = -1$ the region in the first quadrant bounded by the curves $y = 3 - x$ and $y = \dfrac{2}{x}$.

   (A) $\dfrac{7\pi}{3}$ (B) $\pi\left(\dfrac{7}{3} - \pi \ln 4\right)$ (C) $\pi\left(\dfrac{9}{2} + \ln 3\right)$ (D) $\pi\left(\dfrac{10}{3} - 4\ln 2\right)$ (E) 3π

3. The curve defined by the parametric equations $x(t) = \cos t + t \sin t$ and $y(t) = \sin t - t\cos t$, when $0 \le t \le a$ is known to have length $2\pi^2$. Find a.

   (A) $\dfrac{\pi}{4}$ (B) $\dfrac{\pi}{2}$ (C) 2π (D) 4π (E) $\dfrac{4}{\pi^2}$

4. Evaluate $\displaystyle\int_3^\infty \frac{1}{x^2 - 3x + 2} dx$.

   (A) ∞ (B) $-\ln 2$ (C) 0 (D) $\ln 2$ (E) $\ln 3$

5. Evaluate $\displaystyle\int x \cos x dx$.

   (A) $x \sin x + \cos x + C$ (B) $x\cos x - \sin x + C$ (C) $x \sin x$

   (D) $x \sin x - \cos x + C$ (E) $\sin x + C$

6. $\int_0^1 \sqrt{1-x^2}dx =$

(A) $\frac{4-\pi}{4}$ (B) π (C) 0 (D) $\frac{1}{2}\ln 2$ (E) $\frac{\pi}{4}$

7. If the region enclosed by the x-axis, the line $x = 1$, and the curve $y = \sqrt{x}$ is revolved about the vertical line $x = -1$, the volume of the solid generated is

(A) $\frac{32\pi}{15}$ (B) $\frac{16\pi}{3}$ (C) $\frac{16\pi}{15}$ (D) $\frac{8\pi}{15}$ (E) π

8. Which of the following gives the sum of the telescoping series $\sum_{n=1}^{\infty} \frac{2}{n(n+2)}$

(A) $\frac{1}{2}$ (B) $\frac{3}{2}$ (C) 1 (D) 2 (E) $\frac{5}{3}$

9. Find the radius of convergence for the power series $\sum_{n=1}^{\infty} n^3(x-4)^n$

(A) 0 (B) 1 (C) 2 (D) 4 (E) ∞

10. For which values of x does the series $\sum_{n=1}^{\infty} \frac{(x+2)^n}{\sqrt{n}}$ converge?

(A) $-3 < x < -1$ (B) $-3 \leq x < -1$ (C) $-3 \leq x \leq -1$ (D) $-1 \leq x < 1$ (E) $-1 \leq x \leq 1$

11. Find a power series representation for $\frac{x}{2+3x}$ and determine the interval of convergence I.

(A) $\sum_{n=0}^{\infty} \frac{(-1)^n 3^n}{2^{n+1}} x^{n+1}$ with $I = (\frac{-2}{3}, \frac{2}{3})$

(B) $\sum_{n=0}^{\infty} \left(\frac{4}{3}\right)^n x^n$ with $I = (\frac{-3}{4}, \frac{3}{4})$

(C) $\sum_{n=0}^{\infty} \frac{(4^n}{3^{n+1}} x^{n+1}$ with $I = (\frac{-3}{4}, \frac{3}{4})$

(D) $\sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n x^n$ with $I = (\frac{-2}{3}, \frac{2}{3})$

(E) $\sum_{n=1}^{\infty} \frac{(-1)^n 3^{n-1}}{2^n} x^n$ with $I = (\frac{-2}{3}, \frac{2}{3})$

12. Which of the following statements are true?

I. $\cosh^2 x + \sinh^2 x = 1$

II. $\displaystyle\int \frac{1}{\sqrt{25 - x^2}} dx = \frac{1}{5} \arcsin \frac{x}{5} + C$

III. $1 + \tan^2 x = \sec^2 x$

(A) I. only (B) III. only (C) I. and III. only (D) II. and III. only (E) I., II., and III.

13. If $x(t) = t^2 + 1$ and $y(t) = t^3$, then $\dfrac{dy}{dx} =$

(A) $\dfrac{3}{4t}$ (B) $\dfrac{3}{2t}$ (C) $3t$ (D) $6t$ (E) $\dfrac{3}{2}t$

14. Evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \tan x dx$.

(A) $-\infty$ (B) ∞ (C) 0 (D) π (E) $\dfrac{\pi}{2}$

15. Evaluate

$$\lim_{x \to 1} \frac{1 - \frac{1}{x}}{1 - \frac{1}{x^2}}$$

(A) 0 (B) 1 (C) 2 (D) $\dfrac{1}{2}$ (E) does not exist

16. Evaluate

$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{3x}$$

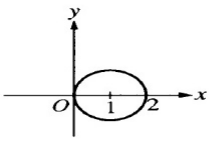
(A) 0 (B) 1 (C) e (D) e^2 (E) e^3

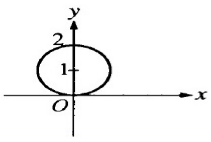
17. Evaluate

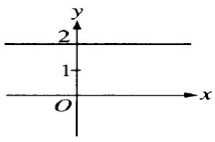
$$\sum_{n=0}^{\infty} 3^{-n}$$

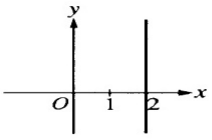
(A) $\dfrac{1}{2}$ (B) $\dfrac{1}{3}$ (C) $\dfrac{2}{3}$ (D) $\dfrac{3}{2}$ (E) diverges

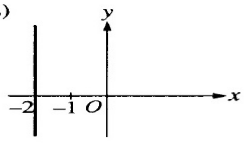
18. Which of the following is the graph of the polar curve $r = 2\cos\theta$?

(A) 

(B) 

(C) 

(D) 

(E) 

19. For $-1 < x < 1$ if $f(x) = \displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^{2n-1}}{2n-1}$, then $f'(x)=$

(A) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}x^{2n}$

(B) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n}x^{2n}$

(C) $\displaystyle\sum_{n=1}^{\infty}(-1)^{2n}x^{2n}$

(D) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n}x^{2n-2}$

(E) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}x^{2n-2}$

20. Which of the following gives the radius of convergence for the power series $\displaystyle\sum_{n=1}^{\infty}\frac{(2x-5)^n}{n^2}$?

(A) 2 (B) 1 (C) $\frac{1}{2}$ (D) 0 (E) ∞

21. Use the Maclaurin series for $g(x) = \cos x$ in this problem. Let $f(x)$ be a function for which $f'(x) = x\cos(x^2)$. What is the coefficient of x^6 in the Maclaurin series for $f(x)$?

(A) $\frac{1}{6!}$ (B) $\frac{1}{6}$ (C) -60 (D) $\frac{-1}{12}$ (E) $\frac{-1}{7!}$

22. For the following list of functions given the Maclaurin series and the interval of convergence.

(a) $f(x) = e^x$

(b) $f(x) = \cos x$

(c) $f(x) = \sin x$

(d) $f(x) = \arctan x$

(e) $f(x) = \ln(1 + x)$

(f) $f(x) = \dfrac{1}{1 + x}$

(g) $f(x) = x \sin x^2$

(h) $f(x) = \displaystyle\int_0^x e^{t^2}\, dt$

(i) $f(x) = \dfrac{1}{1 - x}$

(j) $f(x) = 2 \cosh x$. [Define it first then add term by term.]

23. What is the arc length of $y = \frac{2}{3}x^{3/2}$ from $x = 0$ to $x = 3$.

24. Find the volume of the solid generated by rotating the region bounded by the graphs of $y = \sin x$, $y = 0$, $x = 0$, $x = \pi$ about the x-axis.

25. Evaluate by using Integration by Parts [Hint: twice]

$$\int e^x \sin x dx$$

26. Classify each of the following sequences as absolutely convergent, conditionally convergent, or divergent. Briefly explain why.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n!}$

(b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{(-1)^{n+1}(3n-1)}{2n+5} \right)^n$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{6^n}{(n+1)^n}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{2n!}{(n+1)!}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^4}$

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n+2)}{n(n+1)}$

(g) $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{n^5}$

(h) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n \ln n}$

(i) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^2}$

(j) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(2n)!}{n^n}$

(k) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 2^{4n}}{(2n+1)!}$

(l) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n [2 \cdot 4 \cdot 6 \cdots (2n)]}{2 \cdot 5 \cdot 8 \cdots (3n-1)}$