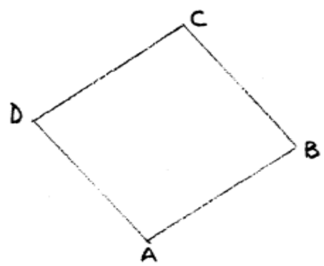
Review Exam 2– MAC 2311H – Fall 2020

1.

A mini-baseball diamond is a square ABCD with side 9 meters. A batter hits the ball at A and runs toward first base B with a speed of 2 m/s. At what rate is his distance from third base D increasing when he is two-thirds of the way to first base?



A. $\frac{3}{\sqrt{13}}$ m/s

B. 2 m/s

C. 4 m/s

D. $\frac{4}{\sqrt{13}}$ m/s

E. $\frac{24}{\sqrt{13}}$ m/s

2.

The derivative of a function g is $g'(x) = \sin x - \sin 2x$, so that $x = 0$ and $x = \pi/3$ are critical numbers of g. Then, g has

A. a local minimum at 0 and a local maximum at $\pi/3$

B. a local maximum at 0 and a local minimum at $\pi/3$

C. a local maximum at 0 and an inflection point at $\pi/3$

D. a local maximum at $\pi/3$

E. inflection points at $0, \pi/3$

3.

The minimum value of $f(x) = 3x + \frac{12}{x^2}$ for $x > 0$ is

A. 6

B. 8

C. $\frac{26}{3}$

D. 9

E. 10

4.

The minute hand on a watch is 2 in long and the hour hand is 1 in long. At two o'clock the distance between the tips of the hands is $\sqrt{3}$ in. How fast is the distance between the tips of the hands decreasing at that moment?

A. $\frac{11\pi}{6}$ in/hour

B. $\frac{11\pi\sqrt{3}}{6}$ in/hour

C. $\frac{11\pi}{12}$ in/hour

D. $\frac{11\pi\sqrt{3}}{12}$ in/hour

E. $\frac{11\pi}{6\sqrt{3}}$ in/hour

5.

Suppose that f is continuous on $[2,5]$ and $2 \le f'(x) \le 5$ for all x in $(2,5)$. Then, the mean value theorem implies that $f(5) - f(2)$ lies in the interval

A. $[6, 15]$
B. $[3, 12]$
C. $[2, 5]$
D. $[0, 5]$
E. $[-5, 5]$

The critical numbers of $R(t) = t^{1/3} - t^{-2/3}$ are

A. 0 and 2
B. -2 only
C. 0 and $\pm\sqrt{3}$
D. -2 and -1
E. 2 and $\pm\sqrt{3}$

6.

A rain gutter is to be constructed from a metal sheet of width 24 cm by bending up one-third of the sheet on each side through an angle θ. In order to choose θ so that the gutter will carry the maximum amount of water, the function to be maximized is

A. $64\,(\cos^2\theta + \cos\theta)$
B. $64\,(\sin\theta\,\cos\theta + \sin\theta)$
C. $32\sin^2\theta + 16\cos^2\theta$
D. $32\,(\sin^2\theta + \sin\theta\,\cos\theta)$
E. $32\cos^2\theta + 16\sin\theta\,\cos\theta$

The total number of local maxima, local minima, and inflection points in the graph of $f(x) = \dfrac{1}{1-x^2}$ is

A. 1
B. 2
C. 3
D. 4
E. 5

7.

8. A box with no top has width twice its height and length four times its height. The material for the sides costs \$6/in^2 and for the base \$4/in^2. If its height is s in. find the rate of change of the cost of the box with respect to s in \$/in.

   a. 52s \$ in b. 62s \$ in c. 104s \$ in d. 208s \$ in e. 86s \$ in

9. If $f(x) = (1 + cos^2 x)^6$ then $f'(\frac{\pi}{4}) =$

   a. $-(\frac{3}{2})^6$ b. $-2(\frac{3}{2})^5$ c. $-4(\frac{3}{2})^6$ d. $4(\frac{3}{2})^6$ e. $-4(\frac{3}{2})^5$

10. A particle moves along the curve $y = \sqrt[3]{11 + x^4}$. As it reaches the point (2, 3), the y-coordinate is increasing at a rate of 32 cm/s. Then, the x-coordinate at that instant is increasing at a rate of

    a. 27 cm/s b. 9 cm/s c. 13.5 cm/s d. 6.75 cm/s e. None of the above

11. The minute hand on a watch is 9 cm long and the hour hand is 4 cm long. How fast, in cm/h , is the distance between the tips of the hands increasing at ten o'clock?

    a. $\frac{1-8\sqrt{3}}{11\sqrt{61}\pi}$ b. $\frac{33\sqrt{3}\pi}{\sqrt{61}}$ c. $\sqrt{61}$ d. $\frac{11\sqrt{61}\pi}{6}$ e. $\frac{6\sqrt{61}}{11\pi}$

12. Assume y is a differentiable function of x. If $\sqrt{xy} = x^2y - 6$, then the slope of the tangent line at the point (1, 9) is
    A. -99/5 B. 40 C. -45 D. -99/2 E. 81/5

13. Two people start from the same point at the same time. One walks north at 2 mi/h and the other walks west at 4 mi/h. How fast is the distance between them changing after 30 minutes?
    A. $20/\sqrt{5}$ mph B. $10/\sqrt{5}$ mph C. $6/\sqrt{5}$ mph D. $5/\sqrt{5}$ mph E. $2/\sqrt{5}$ mph

14. At time 0 a ball is thrown directly upward from a platform 10m tall. Its height above the ground after t seconds is $s(t) = -5t^2 + 5t + 10$, where s is in meters. The ball hits the ground after 2 seconds. What is its velocity at impact?
    A. 0 B. -5 m/s C. -10 m/s D. -15 m/s E. -20 m/s

15. If $g(x) = \tan(\frac{\pi}{2}f(x))$, where $f(0) = 0$ and $f'(0) = 2$, then $g'(0) =$
    A. 4 B. $\pi/2$ C. π D. 2 E. Cannot be determined

16. Use implicit differentiation to find $\frac{dy}{dx}$ at the point (1, 2) if $x^4 - 3x^2y + y^2 + y^3 = 7$.
    A. -2/5 B. 1/2 C. 4/13 D. -3/5 E. 8/13

17. A spherical balloon increases in radius by $\frac{1}{2}$ inch per minute. Find the average rate of change of the volume of the balloon (in^3/minute) when the radius increases from 2 inches to 4 inches (Volume of sphere: $V = \frac{4}{3}\pi r^3$.)

18. Two sides of a triangle are 3 in. and 7 in. and the angle between them is increasing at 0.2 radians per minute. Find the rate at which the area of the triangle is increasing when the angle between the sides is $\frac{\pi}{6}$?

19. A 5 foot ladder standing on level ground leans against a vertical wall. The bottom of the ladder is pulled away from the wall at 2 ft/sec. How fast is the AREA under the ladder changing when the top of the ladder is 4 feet above the ground?

20. The absolute maximum value of $f(x) = \frac{x^2-4}{x^2+2}$ on the interval $[-2, 2]$ is
    a. -4 b. 4 c. 0 d. 2 e. -2

21. Suppose a stone is launched vertically upward with initial velocity 32 ft/sec from a height of 48 ft above ground. After t seconds its height is given by

$$s(t) = -16t^2 + 32t + 48$$

22. At the time when the stone hits the ground it is moving with a speed of
    A. 32 ft/s B. 24 ft/s C. 64 ft/s D. 48 ft/s E. 96 ft/s

23. A bug is moving along the line $y = 2x + 1$. Let $f(t)$ be its distance from the origin at time t. At a certain time t_0, $f'(t_0) = 7$ and $x = 1$. What is $\frac{dx}{dt}$ at that time?
24. Suppose $f(1) = 1$, $f'(1) = 2$, $f''(1) = 3$. If $g(x) = f(x^2)$, then $g''(1) =$
    A. 4 B. 6 C. 12 D. 16 E. 10

25. An inverted conical tank with a height of 8 feet and a radius of 4 feet is drained through a hole in the vertex at a rate of 12 cubic feet per second. What is the rate of change of the water depth when the water depth is 2 feet? (Note: the volume of a cone with base radius r and height h is $\frac{1}{3}\pi r^2 h$.)

26. The intervals on which the function $f(x) = x^4 - 4x^3 + 4x^2$ is decreasing and those on which it is increasing are given by
    A. Increases on $(-\infty, 0) \cup (1, 2)$ and decreases on $(0, 1) \cup (2, \infty)$
    B. Decreases on $(-\infty, 0) \cup (1, 3)$ and increases on $(0, 1) \cup (3, \infty)$
    C. Decreases on $(-\infty, 0) \cup (1, 2)$ and increases on $(0, 1) \cup (2, \infty)$
    D. Decreases on $(-\infty, 2)$ and increases on $(2, \infty)$
    E. None of the above

27. The minimum value of the function $f(x) = \frac{3}{8}x^4 + x^3$ is
    A. -4 B. -3/2 C. -2 D. 0 E. -5/8

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A. $64(\cos^2\theta + \cos\theta)$
B. $64(\sin\theta\cos\theta + \sin\theta)$
C. $32\sin^2\theta + 16\cos^2\theta$
D. $32(\sin^2\theta + \sin\theta\cos\theta)$
E. $32\cos^2\theta + 16\sin\theta\cos\theta$

The total number of local maxima, local minima, and inflection points in the graph of $f(x) = \frac{1}{1-x^2}$ is

A. 1
B. 2
C. 3
D. 4
E. 5

The maximum value of $x^3 - 3x + 9$ for $-3 \le x \le 2$ is

A. 5
B. 7
C. 9
D. 11
E. 13

The minimum value of $x^3 - 3x + 9$ for $-3 \le x \le 2$ is

A. -9
B. -1
C. 3
D. 5
E. 7

Given that $f(3) = 0$ and $f'(x) \geq 3$ for $0 \leq x \leq 3$, the largest $f(0)$ can be is

A. -9

B. -3

C. 0

D. 6

E. Cannot be determined.

If $f'(x) = x(x-1)^2(x-2)$, then f has

A. 3 local minima.

B. 2 local minima and 1 local maximum.

C. 1 local minimum and 2 local maxima.

D. 3 local maxima.

E. 1 local maximum and 1 local minimum.

If $f'(x) = 3(x-1)^{2/3} - x$, the interval(s) where f is concave down is (are)

A. $(-\infty, 9)$ only

B. $(-\infty, 1)$ only

C. $(9, \infty)$ only

D. $(-\infty, 1)$ and $(9, \infty)$

E. $(-\infty, 9)$ and $(9, \infty)$

Given that $f(1) = 9$ and $f'(x) \geq 3$ for $1 \leq x \leq 4$, the smallest $f(4)$ can be is

a. 19

b. 18

c. 12

d. cannot be determined

e. none of the above

28. You are given a function $f(x)$. It could be a polynomial, a rational function, a trigonometric function, a root function, or an arithmetic mix of these.
   (a) Find $f'(x)$ and use $f'(x)$ to find
      (i) Find the critical points of $f(x)$
      (ii) on which intervals is $f(x)$ increasing/decreasing (First Derivative Test).
      (iii) relative extrema.
   (b) Find $f''(x)$ and use it to find
      (i) where $f(x)$ is concave up/down
      (ii) inflection points
      (iii) relative extrema (Second Derivative Test)