The position of a particle moving along a line is given by $s(t) = 2t^3 - 24t^2 + 90t + 7$ for $t \geq 0$. For what values of t is the speed of the particle increasing?

(A) $3 < t < 4$ only
(B) $t > 4$ only
(C) $t > 5$ only
(D) $0 < t < 3$ and $t > 5$
(E) $3 < t < 4$ and $t > 5$

A solid has a rectangular base that lies in the first quadrant and is bounded by the x- and y-axes and the lines $x = 2$ and $y = 1$. The height of the solid above the point (x, y) is $1 + 3x$. Which of the following is a Riemann sum approximation for the volume of the solid?

(A) $\sum_{i=1}^{n} \frac{1}{n}\left(1 + \frac{3i}{n}\right)$

(B) $2\sum_{i=1}^{n} \frac{1}{n}\left(1 + \frac{3i}{n}\right)$

(C) $2\sum_{i=1}^{n} \frac{i}{n}\left(1 + \frac{3i}{n}\right)$

(D) $\sum_{i=1}^{n} \frac{2}{n}\left(1 + \frac{6i}{n}\right)$

(E) $\sum_{i=1}^{n} \frac{2i}{n}\left(1 + \frac{6i}{n}\right)$

Which of the following is the solution to the differential equation $\frac{dy}{dx} = \frac{4x}{y}$, where $y(2) = -2$?

(A) $y = 2x$ for $x > 0$

(B) $y = 2x - 6$ for $x \neq 3$

(C) $y = -\sqrt{4x^2 - 12}$ for $x > \sqrt{3}$

(D) $y = \sqrt{4x^2 - 12}$ for $x > \sqrt{3}$

(E) $y = -\sqrt{4x^2 - 6}$ for $x > \sqrt{1.5}$

6. Let f be the function given by $f(x) = (2x - 1)^5 (x + 1)$. Which of the following is an equation for the line tangent to the graph of f at the point where $x = 1$?

(A) $y = 21x + 2$
(B) $y = 21x - 19$
(C) $y = 11x - 9$
(D) $y = 10x + 2$
(E) $y = 10x - 8$

The rate at which water is sprayed on a field of vegetables is given by $R(t) = 2\sqrt{1 + 5t^3}$, where t is in minutes and $R(t)$ is in gallons per minute. During the time interval $0 \leq t \leq 4$, what is the average rate of water flow, in gallons per minute?

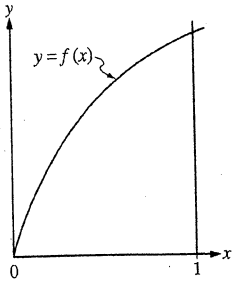
(A) 8.458 (B) 13.395 (C) 14.691 (D) 18.916 (E) 35.833

The radius of a circle is increasing. At a certain instant, the rate of increase in the area of the circle is numerically equal to twice the rate of increase in its circumference. What is the radius of the circle at that instant?

(A) $\frac{1}{2}$ (B) 1 (C) $\sqrt{2}$ (D) 2 (E) 4

A particle moves along the x-axis so that at any time $t > 0$, its velocity is given by $v(t) = 4 - 6t^2$. If the particle is at position $x = 7$ at time $t = 1$, what is the position of the particle at time $t = 2$?

(A) −10 (B) −5 (C) −3 (D) 3 (E) 17



A left Riemann sum, a right Riemann sum, and a trapezoidal sum are used to approximate the value of $\int_0^1 f(x)\,dx$, each using the same number of subintervals. The graph of the function f is shown in the figure above. Which of the sums give an underestimate of the value of $\int_0^1 f(x)\,dx$?

I. Left sum
II. Right sum
III. Trapezoidal sum

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) II and III only

19. Let f be the function given by $f(x) = x^3 - 6x^2$. The graph of f is concave up when

(A) $x > 2$
(B) $x < 2$
(C) $0 < x < 4$
(D) $x < 0$ or $x > 4$ only
(E) $x > 6$ only

If $f(x) = \cos^3(4x)$, then $f'(x) =$

(A) $3\cos^2(4x)$
(B) $-12\cos^2(4x)\sin(4x)$
(C) $-3\cos^2(4x)\sin(4x)$
(D) $12\cos^2(4x)\sin(4x)$
(E) $-4\sin^3(4x)$

$\int (x^3+1)^2\,dx =$

(A) $\frac{1}{7}x^7 + x + C$
(B) $\frac{1}{7}x^7 + \frac{1}{2}x^4 + x + C$
(C) $6x^2(x^3+1) + C$
(D) $\frac{1}{3}(x^3+1)^3 + C$
(E) $\frac{(x^3+1)^3}{9x^2} + C$

$\lim_{x\to\infty} \frac{2x^6+6x^3}{4x^5+3x^3}$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) nonexistent

The functions f and g are differentiable, and $f(g(x)) = x$ for all x. If $f(3) = 8$ and $f'(3) = 9$, what are the values of $g(8)$ and $g'(8)$?

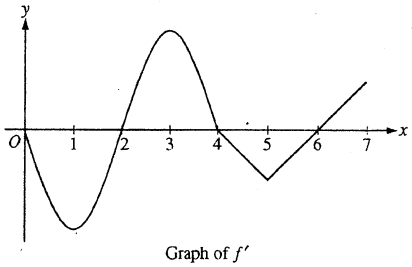
(A) $g(8) = \frac{1}{3}$ and $g'(8) = -\frac{1}{9}$
(B) $g(8) = \frac{1}{3}$ and $g'(8) = \frac{1}{9}$
(C) $g(8) = 3$ and $g'(8) = -9$
(D) $g(8) = 3$ and $g'(8) = -\frac{1}{9}$
(E) $g(8) = 3$ and $g'(8) = \frac{1}{9}$

x	3	4	5	6	7
$f(x)$	20	17	12	16	20

The function f is continuous and differentiable on the closed interval $[3, 7]$. The table above gives selected values of f on this interval. Which of the following statements must be true?

I. The minimum value of f on $[3, 7]$ is 12.
II. There exists c, for $3 < c < 7$, such that $f'(c) = 0$.
III. $f'(x) > 0$ for $5 < x < 7$.

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) I, II, and III



Graph of f'

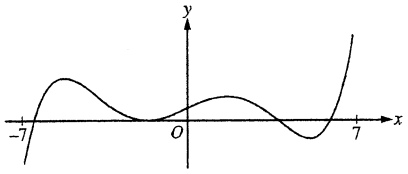
If $f'(x) = (x-2)(x-3)^2(x-4)^3$. then f has which of the following relative extrema?

I. A relative maximum at $x = 2$
II. A relative minimum at $x = 3$
III. A relative maximum at $x = 4$

(A) I only
(B) III only
(C) I and III only
(D) II and III only
(E) I, II, and III

The graph of f', the derivative of the function f, is shown above. On which of the following intervals is f decreasing?

A) $[2, 4]$ only
B) $[3, 5]$ only
C) $[0, 1]$ and $[3, 5]$
D) $[2, 4]$ and $[6, 7]$
E) $[0, 2]$ and $[4. 6]$



Graph of f'

The figure above shows the graph of f', the derivative of the function f, on the open interval $-7 < x < 7$. If f' has four zeros on $-7 < x < 7$, how many relative maxima does f have on $-7 < x < 7$?

(A) One (B) Two (C) Three (D) Four (E) Five

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	−2	−3	4

The table above gives values of the differentiable functions f and g and their derivatives at $x = 1$. If $h(x) = (2f(x) + 3)(1 + g(x))$, then $h'(1) =$

(A) −28 (B) −16 (C) 40 (D) 44 (E) 47

The function f given by $f(x) = 2x^3 - 3x^2 - 12x$ has a relative minimum at $x =$

(A) −1 (B) 0 (C) 2 (D) $\frac{3-\sqrt{105}}{4}$ (E) $\frac{3+\sqrt{105}}{4}$

If $g(x) = x^2 - 3x + 4$ and $f(x) = g'(x)$, then $\int_1^3 f(x)\,dx =$

(A) $-\frac{14}{3}$ (B) −2 (C) 2 (D) 4 (E) $\frac{14}{3}$

x	2.5	2.8	3.0	3.1
$f(x)$	31.25	39.20	45	48.05

The function f is differentiable and has values as shown in the table above. Both f and f' are strictly increasing on the interval $0 \le x \le 5$. Which of the following could be the value of $f'(3)$?

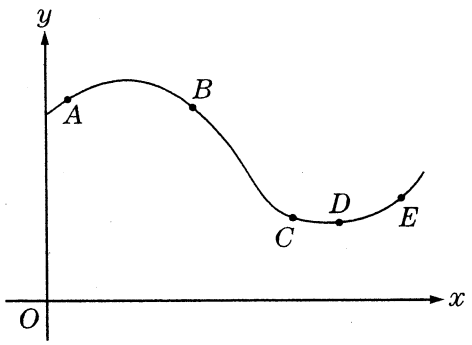
(A) 20 (B) 27.5 (C) 29 (D) 30 (E) 30.5

If f is a continuous function on the closed interval $[a, b]$, which of the following must be true?

(A) There is a number c in the open interval (a, b) such that $f(c) = 0$.
(B) There is a number c in the open interval (a, b) such that $f(a) < f(c) < f(b)$.
(C) There is a number c in the closed interval $[a, b]$ such that $f(c) \ge f(x)$ for all x in $[a, b]$.
(D) There is a number c in the open interval (a, b) such that $f'(c) = 0$.
(E) There is a number c in the open interval (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.

At which of the five points on the graph in the figure at the right are $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ both negative?

(A) A
(B) B
(C) C
(D) D
(E) E



What is $\lim_{x\to\infty} \frac{x^2-4}{2+x-4x^2}$?

(A) -2
(B) $-\frac{1}{4}$
(C) $\frac{1}{2}$
(D) 1
(E) The limit does not exist.

Let $u(t)=\frac{t^2+4t-21}{t^2-9}$. Assume also that $u(t)$ is continuous for all positive real numbers. Determine $\lim_{t\to 3} u(t)$.

(A) $\frac{5}{3}$
(B) 3
(C) $-\frac{10}{3}$
(D) 0
(E) This is not possible. The function cannot be continuous at $t=3$.

Which of the following statements about the function given by $f(x) = x^4 - 2x^3$ is true?

(A) The function has no relative extremum.
(B) The graph of the function has one point of inflection and the function has two relative extrema.
(C) The graph of the function has two points of inflection and the function has one relative extremum.
(D) The graph of the function has two points of inflection and the function has two relative extrema.
(E) The graph of the function has two points of inflection and the function has three relative extrema.

What is the average rate of change of the function f given by $f(x) = x^4 - 5x$ on the closed interval $[0, 3]$?

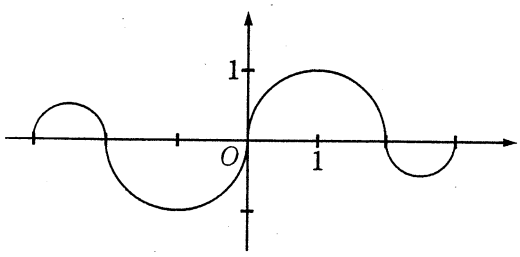
(A) 8.5
(B) 8.7
(C) 22
(D) 33
(E) 66

The slope of the tangent to the curve $y^3x + y^2x^2 = 6$ at $(2, 1)$ is

(A) $-\frac{3}{2}$
(B) -1
(C) $-\frac{5}{14}$

A spherical balloon is being inflated at a rate of 3 cubic inches per second. Determine the rate of change of the radius of the balloon ıen the balloon's radius is 5 inches, curate to three decimal places. The ılume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

ı) 3.000 inches per second
ɜ) 1.667 inches per second
ɔ) 0.010 inches per second
ɔ) −2.000 inches per second
ɜ) 0.120 inches per second



Graph of f

he graph of the function f above consists of four semicircles. If $g(x) = \int_0^x f(t)\,dt$, here is $g(x)$ nonnegative?

) $[-3, 3]$
) $[-3, -2] \cup [0, 2]$ only
) $[0, 3]$ only
) $[0, 2]$ only
) $[-3, -2] \cup [0, 3]$ only

If f is differentiable at $x = a$, which of the following could be false?

(A) f is continuous at $x = a$.
(B) $\lim_{x\to a} f(x)$ exists.
(C) $\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$ exists.
(D) $f'(a)$ is defined.
(E) $f''(a)$ is defined.

The diameter of a circle is increasing at a rate of 5 cm/sec. At what rate is the area increasing when the diameter is 10 cm?

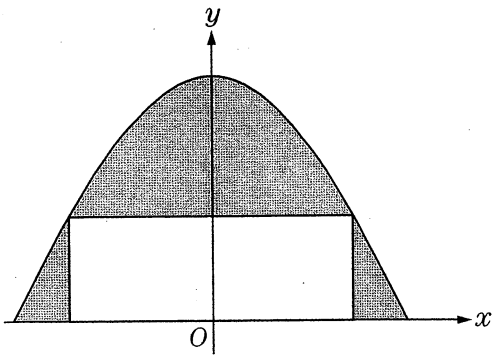
(A) 6.25π cm²/sec.
(B) 25π cm²/sec.
(C) 50π cm²/sec.
(D) $125\pi/3$ cm²/sec.
(E) 100π cm²/sec.

If $f(x) = \sin^2(3 - x)$, then $f'(0) =$

(A) $-2 \cos 3$
(B) $-2 \sin 3 \cos 3$
(C) $6 \cos 3$
(D) $2 \sin 3 \cos 3$
(E) $6 \sin 3 \cos 3$

$\int (x-1)\sqrt{x}\, dx =$

(A) $\frac{3}{2}\sqrt{x} - \frac{1}{\sqrt{x}} + C$
(B) $\frac{2}{3}x^{3/2} + \frac{1}{2}x^{1/2} + C$
(C) $\frac{1}{2}x^2 - x + C$
(D) $\frac{2}{5}x^{5/2} - \frac{2}{3}x^{3/2} + C$
(E) $\frac{1}{2}x^2 + 2x^{3/2} - x + C$



The cost of producing x units of a certain item is $c(x) = 2000 + 8.6x + 0.5x^2$. What is the average rate of change of c with respect to x when the level of production increases from $x = 300$ to $x = 310$ units?

(A) 313.6
(B) 310.0
(C) 214.2
(D) 200.0
(E) 10

A rectangle with one side on the x-axis has its upper vertices on the graph of $y = \cos x$, as shown in the figure above. What is the minimum area of the shaded region?

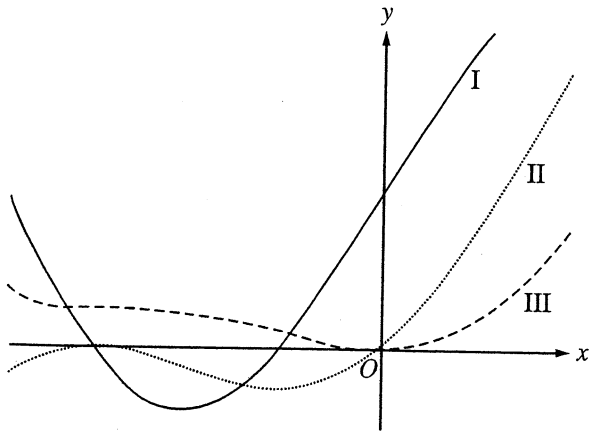
(A) 0.799
(B) 0.878
(C) 1.140
(D) 1.439
(E) 2.000

Given $f(x) = \sin x + \cos x$ on the interval $[0,\pi]$, what is the absolute maximum function value?

(A) $-\sqrt{2}$
(B) -1
(C) 0
(D) 1
(E) $\sqrt{2}$

If $3x^2 + 2xy + y^2 = 1$, then $\frac{dy}{dx} =$

(A) $-\frac{3x + y}{y^2}$
(B) $-\frac{3x + y}{x + y}$
(C) $\frac{1 - 3x - y}{x + y}$
(D) $-\frac{3x}{1 + y}$
(E) $-\frac{3x}{x + y}$



Three graphs labeled I, II, and III are shown above. One is the graph of f, one is the graph of f', and one is the graph of f''. Which of the following correctly identifies each of the three graphs?

	f	f'	f''
(A)	I	II	III
(B)	I	III	II
(C)	II	I	III
(D)	II	III	I
(E)	III	II	I

If $\frac{d}{dx}f(x) = g(x)$ and if $h(x) = x^2$, then $\frac{d}{dx}f(h(x)) =$

(A) $g(x^2)$
(B) $2xg(x)$
(C) $g'(x)$
(D) $2xg(x^2)$
(E) $x^2g(x^2)$

A showroom is to be made in the shape of a rectangle. The perimeter of the showroom must be 1,200 feet. What is the maximum area the showroom can have?

(A) 22,500 square feet
(B) 25,000 square feet
(C) 90,000 square feet
(D) 200,000 square feet
(E) 250,000 square feet

A gun is fired vertically upward from a position 100 feet above ground at an initial velocity of 400 feet per second. Determine the maximum height of the projectile. The acceleration of gravity is $-32\frac{ft}{sec^2}$.

(A) 3000 feet
(B) 2600 feet
(C) 2200 feet
(D) 1800 feet
(E) 1400 feet

If $f(x) = 6x^{3/2}$, then $f''(16) =$

(A) 9/16
(B) 9/8
(C) 36
(D) 144
(E) 384

$-\sin^2 x + 4 - \cos^2 x =$

(A) –5
(B) –3
(C) 3
(D) 5
(E) cannot be determined

The table below provides data points for the continuous function $y = h(x)$.

x	0	2	4	6	8	10
$h(x)$	9	25	30	16	25	32

Use a right Riemann sum with 5 subdivisions to approximate the area under the curve of $y = h(x)$ on the interval [0, 10].

(A) 256
(B) 235
(C) 210
(D) 206
(E) 242

The slope of $9x - 4x \ln y = 3$ at (1/3,1) is

(A) $9 - 4 \ln 3$
(B) 5
(C) 6
(D) 27/4
(E) $9 + 4 \ln 3$

Given $y = \sqrt{x^3}$, what is $y'''(4)$?

(A) –4
(B) –3/64
(C) –1/48
(D) 1/6
(E) 4

$$\lim_{x \to 0} \frac{2\sin x \cos x}{2x}$$

(A) –2
(B) –1
(C) undefined
(D) 1/2
(E) 1

If $y = (x + 5)(3x)$, what is y'?

(A) $3x + 15$
(B) 3
(C) 4
(D) $6x + 15$
(E) $3 + 3x$

Where does the graph of $f(x) = 3x^2 - 24x + 36$ change direction?

(A) $x = 2$ and $x = 6$
(B) $x = 2$
(C) $x = 6$
(D) $x = 4$ and $x = 6$
(E) $x = 4$

What is the equation of the line that is tangent to the curve $M(t) = -t^2 + 2t + 4$ at the point (–1,1)?

(A) $M = t + 1$
(B) $M = 3t/5 + 52/5$
(C) $M = t + 2$
(D) $M = 3t/5 - 2$
(E) $M = 4t + 5$

Find the values of x for which the function

$$f(x) = x^6 + 3x^5 - \frac{15}{2}x^4 - 40x^3 - 60x^2 + 8x + 5$$

has inflection points. Hint:

$$x^4 + 2x^3 - 3x^2 - 8x - 4 = (x^2 - 4)(x^2 + 2x + 1).$$

(A) f has no inflection points
(B) $x = -2, 2$
(C) $x = -1, 0, 1$
(D) $x = -2, -1, 2$
(E) $x = 0$

$$\lim_{x \to 2} \frac{4x^2 - 16}{(x - 2)} =$$

(A) –3
(B) –1/4
(C) –1
(D) 0
(E) 16

On the interval $(0, 2\pi)$, the function $f(x) = \sin(x)\cos(x)$ has critical points at:

(A) $x = \frac{\pi}{4}, \frac{5\pi}{4}$
(B) $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$
(C) $x = \frac{3\pi}{4}, \frac{7\pi}{4}$
(D) $x = \frac{\pi}{2}, \pi, \frac{3\pi}{2}$
(E) There are no critical points for this function on the given interval.

What is the slope of the tangent line to the curve $y = \cos^2(3x)$ at the point $(\pi/4, 1/2)$?

(A) –3
(B) 1/2
(C) 0
(D) 2
(E) 3

$$\lim_{x \to 0} \frac{-\cos x + 1 - \sin x}{x}$$

(A) –1
(B) 0
(C) undefined
(D) 1
(E) ∞

The Panda Bear Club realizes a monthly revenue of $R(x) = 5{,}250x - 10x^2$ dollars per month when the fee per person is x dollars. What is the marginal revenue when the fee is $21 per person?

(A) –$20
(B) $4,830
(C) $61,560
(D) $105,840
(E) $1,126,755

Where is the function $f(x) = 5/(x^2 - 2x - 15)$ discontinuous?

(A) $x = -5$ and $x = -3$
(B) $x = -5$ and $x = 3$
(C) $x = -3$ and $x = 5$
(D) $x = 3$ and $x = 5$
(E) $x = 2$ and $x = 15$

$y' = \dfrac{4x}{y}$ and $x = -1$ when $y = 4$. What can x be when $y = 6$?

(A) –6
(B) $-\sqrt{6}$
(C) –2
(D) 2
(E) 6

If the function f is continuous for all real numbers and if $f(x) = (2x^2 + x - 15)/(x + 3)$ when $x \neq -3$, then $f(-3) =$

(A) –15
(B) –11
(C) –3
(D) 0
(E) 5/2

If $g(x) = \csc x - \cot x$, then $g'(\pi/6) =$

(A) $4 - 2\sqrt{3}$
(B) $2 - 2\sqrt{3}$
(C) $2 - \sqrt{3}$
(D) 1
(E) $4 + \sqrt{3}$

If $g(x) = 8x^3 - 3x^2 + 4$, then a relative minimum occurs at

(A) (–1/2,9/4)
(B) (0,4)
(C) (1/8,127/32)
(D) (1/4,63/16)
(E) (0,4) and (1/4,63/16)

If $f(x) = 5/(x^3 - 1)$ and $g(x) = 2x$, then $f(g(-1)) =$

(A) –5/2
(B) –2
(C) –5/9
(D) 5/6
(E) undefined

If $s(x) = \sin^2 x$, then $s''(x) =$

(A) –2
(B) $-2 \cos x \sin x$
(C) $2 \sin x \cos x$
(D) $2\cos^2 x - 2\sin^2 x$
(E) 2

$$\lim_{x \to \infty} \frac{3x^2 - 5x + 4}{6x^2 + 7x - 1}$$

(A) –4
(B) –5/7
(C) 0
(D) 1/6
(E) 1/2

If $h(x) = (6x - 3x^2)/(4 + x)$, then $h'(x) =$

(A) $6 - 6x$
(B) $3/2 - 3x$
(C) –3
(D) $(-3x^2 - 24x - 24)/(4 + x)^2$
(E) $[-3(x^2 + 8x - 8)]/(x + 4)^2$