Name: ______________________

Final Exam – Math 2311H – Spring 2014

Directions: remember to show work in the free response part.

1. $\int_0^2 (4x^3 - 6x)dx =$

   (A) 2 (B) 4 (C) 6 (D) 36 (E) 42

2.

$$f(x) = \begin{cases} x^2+1 & -1 \le t < 1 \\ -x+1 & 1 \le x < 2 \\ -1 & x > 2 \end{cases}$$

   The points in the domain of f at which f is continuous are

   (A) $(-1,1) \cup (1,\infty)$ (B) $[-1,1) \cup (1,\infty)$ (C) $(-1,1) \cup (1,2) \cup (2,\infty)$

   (D) $[-1,1) \cup (1,2) \cup (2,\infty)$ (E) $(-1,2) \cup (2,\infty)$

3. If $f(x) = x\sqrt{2x-3}$, then $f'(x) =$

   (A) $\frac{3x-3}{\sqrt{2x-3}}$ (B) $\frac{x}{\sqrt{2x-3}}$ (C) $\frac{1}{\sqrt{2x-3}}$ (D) $\frac{-x-3}{\sqrt{2x-3}}$ (E) $\frac{5x-6}{2\sqrt{2x-3}}$

4. If $f(x) = (x-1)^{\frac{3}{2}} + \frac{e^{x-2}}{2}$ then $f'(2) =$

   (A) 1 (B) $\frac{3}{2}$ (C) 2 (D) $\frac{7}{2}$ (E) $\frac{3+e}{2}$

5. The graph of $f(x) = 3x^4 = 16x^3 + 24x^2$ is concave down for

   (A) $(-\infty, 0)$ (B) $(0,\infty)$ (C) $(-\infty,-2) \cup (-\frac{2}{3},\infty)$

   (D) $(-\infty,\frac{2}{3}) \cup (2,\infty)$ (E) $(\frac{2}{3},2)$

6. $\frac{d}{dx}\cos^2(x^3) =$

(A) $6x^2 \sin(x^3)\cos(x^3)$ (B) $6x^2\cos(x^3)$ (C) $\sin^2(x^3)$

(D) $-6x^2\sin(x^3)\cos(x^3)$ (E) $-2\sin(x^3)\cos(x^3)$

7. What are the values of x for which the function $f(x) = (x^2-3)e^{-x}$ is decreasing?

(A) all reals (B) $(-\infty,-1)\cup(3,\infty)$ (C) $(-3,1)$ (D) $(-1,3)$ (E) no values

8. Let f be a function such that $\lim_{\Delta x\to 0}\frac{f(2+\Delta x)-f(2)}{\Delta x} = 5$. Which of the following statements must be true?

I. f is continuous at $x = 2$

II. f is differentiable at $x = 2$

III. $\lim_{x\to 2} f(x) = 5$.

(A) I. only (B) II. only (C) I. and II. only

(D) I. and III. only (E) II. and III. only

9. If $y = xy + x^2 - 1$, then $x = -1$, $\frac{dy}{dx} =$ [Hint: use equation to find y.]

(A) $\frac{1}{2}$ (B) $\frac{1}{2}$ (C) 1 (D) -1 (E) -2

10. The closed interval $[a,b]$ is partitioned into n equal subintervals, each of width Δx, by the numbers, $a_0 = a, a_1, \ldots, a_{n-1}, a_n = n$ where $a_i = a + i\Delta x$. What is $\lim_{n\to\infty}\sum_i^n = 1\sqrt{a_i}\Delta x$?

(A) $\frac{2}{3}\left(b^{\frac{3}{2}} - a^{\frac{3}{2}}\right)$ (B) $b^{\frac{3}{2}} - a^{\frac{3}{2}}$ (C) $\frac{3}{2}\left(b^{\frac{3}{2}} - a^{\frac{3}{2}}\right)$

(D) $b^{\frac{3}{2}} - a^{\frac{3}{2}}$ (E) $2\left(b^{\frac{3}{2}} - a^{\frac{3}{2}}\right)$

11. $\lim_{x \to 0} \frac{\sin 5x}{x} =$

(A) 0 (B) $\frac{1}{5}$ (C) 1 (D) 5 (E) DNE

12. Let f be a continuous function on the closed interval $[-3, 6]$. If $f(x-3) = -1$ and $f(6) = 3$, then the Intermediate Value Theorem guarantees that

(A) $f(0) = 0$

(B) $f'(c) = \frac{4}{9}$ for at least one $c \in [-3, 6]$

(C) $-1 \leq f(x) \leq 3$ for all $x \in [-3, 6]$

(D) $f(c) = 1$ for at least one $c \in [-3, 6]$

(E) $f(c) = 0$ for at least one $c \in [-1, 3]$.

13. What is the average value of $f(x) = x^2\sqrt{x^3+1}$ over the interval $[0, 2]$?

(A) $\frac{26}{9}$ (B) $\frac{13}{3}$ (C) $\frac{26}{3}$ (D) 13 (E) 26

14. Which of the following statements are true?

I. If the continuous function f has a relative extrema on the open interval (a, b), then $f'(c) = 0$.

II. If the continuous function f is concave up on $[a, b]$, then the Trapezoidal Riemann Sum will yield an over approximation of the actual area.

III. If f is continuous on the open interval I containing a, then the function $F(x) = \int_a^x f(t)dt$ is differentiable on I.

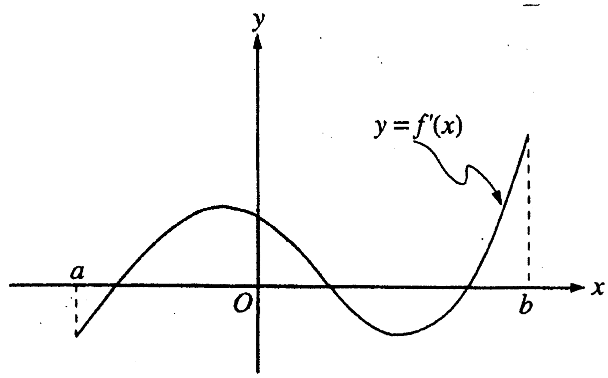
(A) II. only (B) III. only (C) II. and III. only

(D) I. and II. only (E) I., II. and III.

15. The acceleration of a particle moving along the x-axis at time t is given by $a(t) = 6t + 2$. If the velocity is 5 when $t = 1$ and the position is 1 when $t = 0$, then the position function is given by $p(t) =$

(A) $9t^2 + 1$ (B) $3t^3 + t^2 + 1$ (C) $t^3 + t^2 + 1$

(D) $t^3 + t^2 + 5t + 1$ (E) $9t^3 - 4t^2 - 77t + 55$



16. The graph of f', the derivative of f, is shown in the figure above. Which of the following describes all relative extrema of f on the open interval (a, b)?

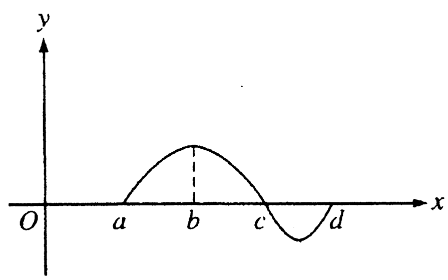
(A) One relative maximum and two relative minima

(B) Two relative maxima and one relative minimum

(C) Three relative maxima and one relative minimum

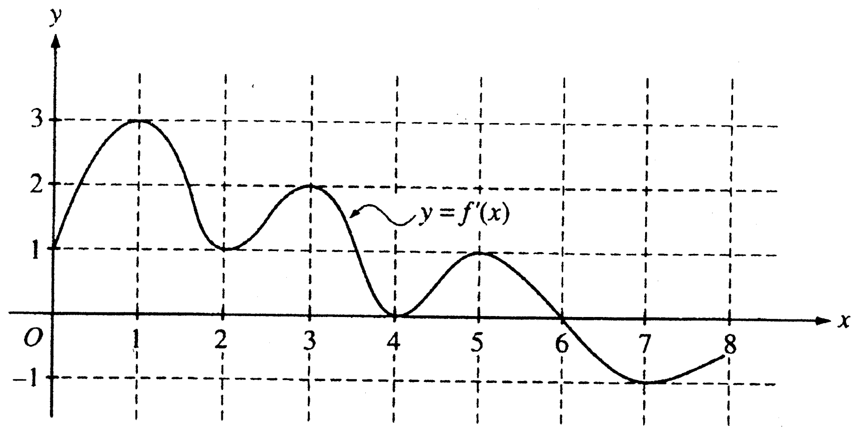
(D) One relative maximum and three relative minima

(E) Three relative maximum and two relative minima



17. The graph of f is shown in the figure above. If $g(x) = \int_a^x f(t)dt$, for what value of x does $g(x)$ have a maximum?

(A) a (B) b (C) c (D) d (E) none



The function f is defined on the closed interval $[0, 8]$. The graph of the derivative f' is shown above.

18. The point $(3, 5)$ is on the graph of $y = f(x)$. An equation of the line tangent to the graph of f at $(3, 5)$ is

(A) $y = 2$

(B) $y = 5$

(C) $y - 5 = 2(x - 3)$

(D) $y + 5 = 2(x - 3)$

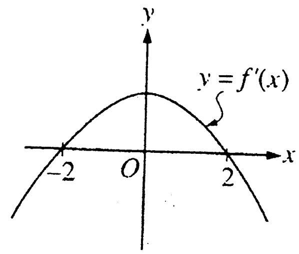
(E) $y + 5 = 2(x - 3)$

19. How many points of inflection does the graph of f have?

(A) Two (B) Three (C) Four (D) Five (E) Six

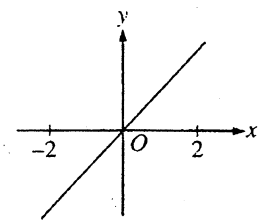
20. At what value of x does the absolute minimum of f occur?

(A) 0 (B) 2 (C) 4 (D) 6 (E) 8

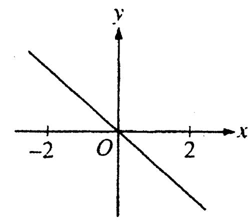


21. The graph of the <u>derivative</u> of f is shown above. Which of the following could be the graph of f?

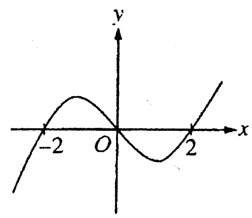
(A)



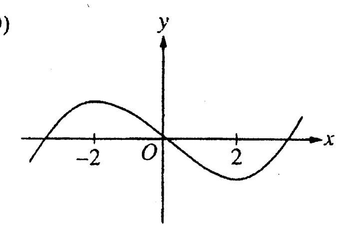
(B)



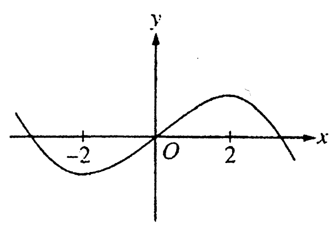
(C)

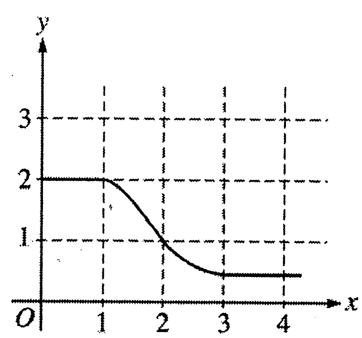


(D)



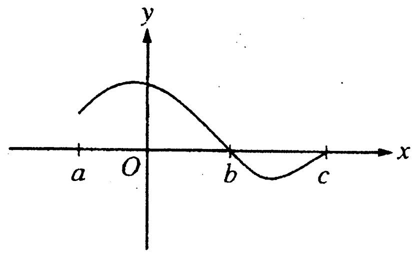
(E)





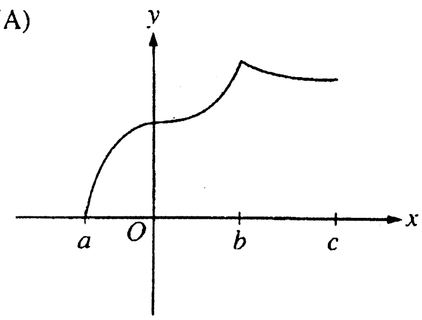
22. The graph of f is shown in the figure above. if $\int_1^3 f(x)dx = 2.3$ and $F'(x) = f(x)$, then $F(3) - F(0)=$

(A) 0.3 (B) 1.3 (C) 3.3 (D) 4.3 (E) 5.3

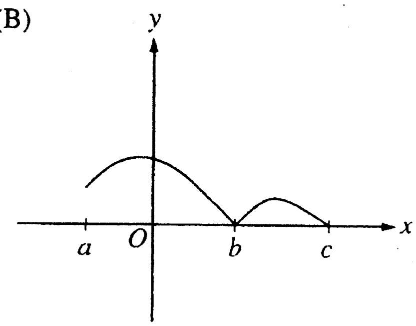


23. Let $f(x) = \int_a^x h(t)dt$, where $h(t)$ has the graph shown above. Which of the following could be the graph of f?

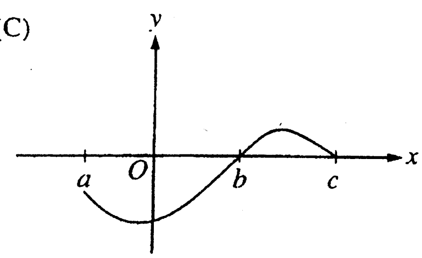
(A)



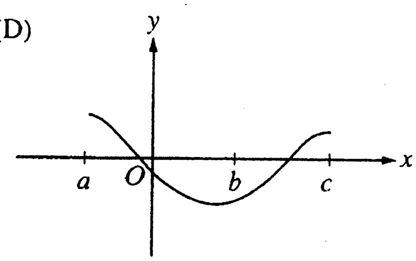
(B)



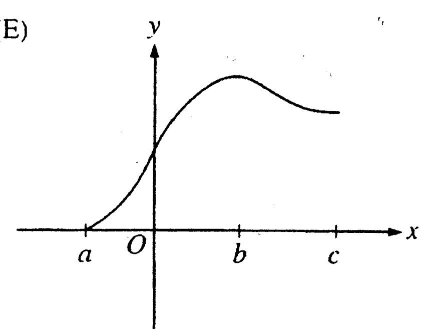
(C)



(D)



(E)



24. State the definition for the following symbol

$$\lim_{x \to a} f(x) = L.$$

25. Use the limit process to evaluate the limit (e.g. use algebra). Show work.

$$\lim_{x \to 3} \frac{\sqrt{x+1} - 2}{x}$$

26. Use a trig identity to compute $\lim_{\theta \to 0} \dfrac{1 - \cos\theta}{2\sin^2\theta}$

27. In the figure below, a 40ft. ladder is sliding down the wall. Label the distance from the wall to the foot of the ladder at instant t by $x(t)$ and the height of the ladder at instant t by $h(t)$. What is $x(t)$ at the instant when the rate of change of $x(t) = \frac{3}{4}$ the rate of change of $y(t)$? (Hint: $\frac{dy}{dt}$ is negative and you should be able to solve y in terms of x.]

28. The gravitational force due to gravity on the planet Jupiter is about $80 ft/s^2$.

(a) Construct the height function of a free falling object on Jupiter using that $a(t) = -80$. Let v_0 denote the initial velocity and h_0 denote the initial height.

(b) If a boulder falls off a 1,000ft high cliff (on Jupiter) how long is it in the air before it hits the ground?

29. In this problem we will calculate the integral $\int_0^1 (x^2+1)dx$ using a Right Riemann Sum.

(a) Let $n = 3$. Split the interval into n equal parts. What is the length of each equal part: $\Delta x = ?$

(b) Find the endpoints of each subinterval: a_0, a_1, a_2, a_3.

(c) Find the Right Riemann Sum: $R = \sum_{i=1}^{3} f(a_i)\Delta x$.

Next we find the total area.

(d) For a fixed n what is Δx? For each $i = 1, 2, \ldots, n$ find a_i and $f(a_i)$.

(e) Find the Right Riemann Sum (in terms of n): $R(n) = \sum_{i=1}^{n} f(a_i)\Delta x$.

(f) Use that $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ to convert your answer in (e).

(g) Find $\int_0^1 (x^2+1)dx = \lim_{n\to\infty} R(n)$.

30. Let $f(x) = \ln(x^2 + 1)$. Find $f'(x)$ and $f''(x)$. Graph the function. Use all three of these to find the following:

(a) critical points of $f(x)$,

(b) where $f(x)$ is increasing/decreasing,

(c) relative extrema

(d) where $f(x)$ is concave up/down

(e) inflection points

(f) limits at infinity (i.e. horizontal asymptotes)

(g) absolute extrema

31. Let $f(x) = \dfrac{\cos x}{\sqrt{1+\sin x}}$.

(a) Use substitution to find $\displaystyle\int \frac{\cos x}{\sqrt{1+\sin x}}dx$.

(b) Use your answer in (a) to find $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{1+\sin x}}dx$. (Points will be taken off if you use your calculator only.)

32. Let $f(x) = \dfrac{2x+1}{x+4}$. Use a change of variable and substitution to find $\displaystyle\int \frac{2x+1}{x+4}dx$. [Hint: use $u = x+4$.]