Name: ______________________

Final Exam – MAC 2311H – Fall 2013

Directions: make sure to show work or explain how you got an answer. If using a graph from your calculator explain what steps you took to get the graph.

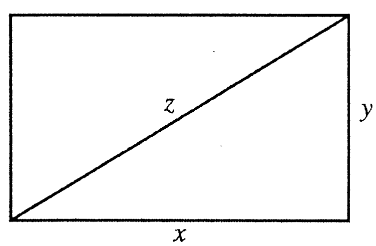
1. A particle moves along the x-axis so that at any time $t \geq 0$, its velocity is given by $v(t) = \sin(2t)$. If the posisiton of the particle at $t = \frac{\pi}{2}$ is $x = 4$, what is the particle's position at time $t = 0$?

   (A) $-\frac{1}{2}$ (B) 2 (C) 3 (D) 5 (E) 8

x	0	2	4	6
$f(x)$	4	k	8	12

2. The function f is continuous on the closed interval $[0,6]$ and has the values given in the table above. The trapezoidal approximation for $\int_0^6 f(x)dx$ found with 3 subintervals of equal length is 52. What is the value of k?

   (A) 2 (B) 6 (C) 7 (D) 10 (E) 14



3. The sides and diagonal of the rectangle above are strictly increasing with time. At the instant when $x = 4$ and $y = 3$, $\frac{dx}{dt} = \frac{dz}{dt}$ and $\frac{dy}{dt} = k\frac{dz}{dt}$. What is the value of k at that instant?

   (A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) 3 (D) 4 (E) 4π

4. If $f'(x) = \frac{2}{x}$ and $f(e^{1/2}) = 5$, then $f(e)$=

   (A) 2 (B) $\ln 25$ (C) $5 + \frac{2}{e} - \frac{2}{e^2}$ (D) 6 (E) 25

5. What is the slope of the line tangent to the curve $y + 2 = \frac{x^2}{2} - 2\sin y$ at the point $(2, 0)$?

(A) -2 (B) 0 (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) 2

6. If $f(x) = \arccos(x^2)$, then $f'(x) =$

(A) $\frac{1}{\sqrt{1-x^4}}$ (B) $\frac{-2x}{\sqrt{1-x^4}}$ (C) $\frac{2x}{\sqrt{1-x^4}}$

(D) $\frac{-4x^3}{\sqrt{1-x^4}}$ (E) $\frac{4x^3}{\sqrt{1-x^4}}$

7. If f is the function given by $f(x) = \int_4^{2x} \sqrt{t^2 - t}dt$, then $f'(2)$=

(A) 0 (B) $\frac{7}{2\sqrt{12}}$ (C) $\sqrt{2}$ (D) $\sqrt{12}$ (E) $2\sqrt{12}$

8. If $\int_0^1 f(x)dx = 2$ and $\int_0^4 f(x)dx = -3$, then $\int_1^4 (3f(x) + 2)dx =$

(A) -13 (B) -9 (C) -7 (D) 3 (E) 21

9. If f is a continuous function on the closed interval $[a, b]$, which of the following must be true?

I. There is a number c in the open interval (a, b) such that $f(c) = 0$.
II. There is a number c in the closed interval $[a, b]$ such that $f(c) \geq f(x)$ for all $x \in [a, b]$.
III. There is a number c in the open interval $[a, b]$ such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.

(A) I. and II. only (B) III. only (C) II. only

(D) I., II., and III. (E) II. and III. only

10. A gun is fired vertically upward from a position 100ft. above the ground at an initial velocity of $400 ft/s$. Determine the maximum height of the projectile. (The acceleration of gravity is -32 ft/s^2.

(A) 3000 ft (B) 2600 ft (C) 2200ft (D) 1800ft (E) 1400ft

11. If $f(x) = x^3 + 3x^2 + kx + 4$ has a horizontal tangent and a point of inflection at the same value of x, what is the value of k?

(A) 0 (B) 1 (C) -1 (D) -3 (E) 3

12. $\lim_{n\to\infty} \frac{1}{n}[(\frac{1}{n})^2 + (\frac{2}{n})^2 + \cdots + (\frac{n-1}{n})^2] =$

(A) $\int_0^1 \frac{1}{x^2}dx$ (B) $\int_0^1 x^2 dx$ (C) $\int_0^1 \frac{2}{x^2}dx$

(D) $\int_0^1 \frac{1}{x}dx$ (E) $\int_0^2 x^2 dx$

13. The graph of $f(x) = 5x^4 - x^5$ has a point of inflection at which value(s) of x?

(A) $x = 0$ only (B) $x = 3$ only (C) $x = 4$ only

(D) $x = 0$ and $x = 3$ only (E) $x = 0$ and $x = 4$ only

14. If $f(x) = 2 + |x - 3|$ for all x, then the value of the derivative $f'(x)$ at $x = 3$ is

(A) -1 (B) 0 (C) 1 (D) 2 (E) does not exist

15. If $\sin x = e^y$, $0 < x < \pi$, what is $\frac{dy}{dx}$ in terms of x?

(A) $-\tan x$ (B) $-\cot x$ (C) $\cot x$ (D) $\tan x$ (E) $\csc x$

16. What is the average value of $3x^2 - x^2$ over the interval $-1 \leq t \leq 2$?

(A) $\frac{11}{4}$ (B) $\frac{7}{2}$ (C) 8 (D) $\frac{33}{4}$ (E) 16

17. The Mean Value Theorem guarantees the existence of a special point on the graph of $y = \sqrt{x}$ between $(0, 0)$ and $(4, 2)$. The x-coordinate of this point is?

(A) $x = 2$ (B) $x = 1$ (C) $x = \frac{1}{4}$ (D) $x = \frac{1}{2}$ (E) $x = \frac{1}{9}$

18. If $h(x) = f(x)^2 - g(x)^2$, $f'(x) = -g(x)$, and $g'(x) = f(x)$, then $h'(x) =$

(A) 0 (B) 1 (C) $-4f(x)g(x)$ (D) $(-g(x))^2 - f(x)^2$ (E) $-2(-g(x) + f(x))$

19. If $F(x) = \int_0^x e^{-t^2}\,dt$, then $F'(x) =$

(A) $2xe^{-x^2}$ (B) $-2xe^{-x^2}$ (C) $\dfrac{e^{-x^2+1}}{-x^2+1}$ (D) $e^{-x^2} - 1$ (E) e^{-x^2}

20. $\int (x^3 - 3x)dx =$

(A) $3x^2 - 3 + C$ (B) $4x^4 - 6x^2 + C$ (C) $\dfrac{x^4}{3} - 3x^2 + C$

(D) $\dfrac{x^4}{4} - 3x + C$ (E) $\dfrac{x^4}{4} - \dfrac{3x^2}{2} + C$

21. Given the function defined by $f(x) = 3x^5 - 20x^3$, find all values of x for which the graph of f is concave up.

(A) $x > 0$ (B) $-\sqrt{2} < x < 0$ or $x > \sqrt{2}$ (C) $-2 > x > 0$ or $x > 2$

(D) $x > \sqrt{2}$ (E) $-2 < x < 2$

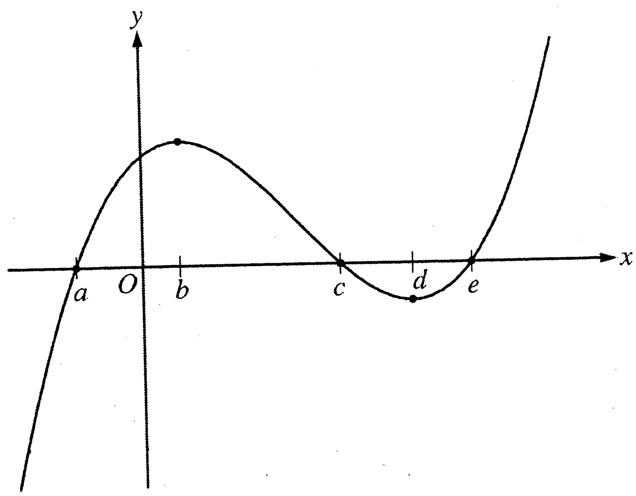
22. If $f(x) = x + \dfrac{1}{x}$ then the set of values for which f increases is

(A) $(-\infty, -1) \cup (1, \infty)$ (B) $(-1, 1)$ (C) $(-\infty, \infty)$

(D) $(0, \infty)$ (E) $(-\infty, 0) \cup (0, \infty)$

23. If $y = \ln(x^2 + y^2)$, then the value of $\dfrac{dy}{dx}$ at the point $(1, 0)$ is

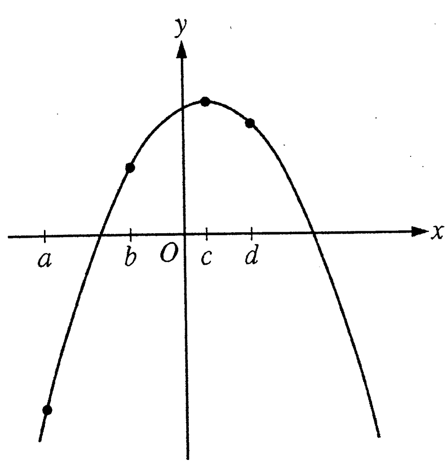
(A) 0 (B) $\dfrac{1}{2}$ (C) 1 (D) 2 (E) undefined



Graph of f

24. The figure above shows the graph of the polynomial function f. For which value of x is it true that $f''(x) < f'(x) < f(x)$?

(A) a (B) b (C) c (D) d (E) e

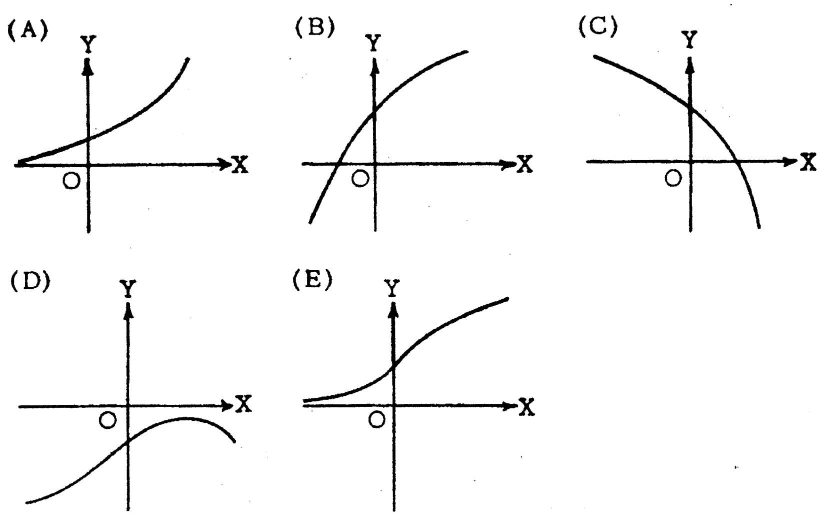


Graph of f

25. The figure above shows the graph of a function f. Which of the following has the greatest value?

(A) $f(a)$ (B) $f'(a)$ (C) $f'(c)$ (D) $f(c) - f(d)$ (E) $\dfrac{f(b) - f(a)}{b - a}$

26. If y is a function of x such that $y' > 0$ for all x and $y'' < 0$ for all x, which of the following could be part of the graph of $y = f(x)$?



27. Estimate the area under the curve $f(x) = x^4$ for $0 \leq x \leq 1$ by computing the Right-Hand Riemann Sum with $n = 4$. Find Δx. Is your final answer an over or under estimate?

28. Define the following hyperbolic functions:

(a) $\cosh x$

(b) $\sinh x$

29. State the derivatives of the following functions. You **do not** need to compute the limit of a difference quotients.

(a) $f(x) = \arctan(3x^2)$

(b) $g(x) = \cos(e^x)$

(c) $h(x) = \frac{x^2+1}{xe^x}$

(d) $k(x) = \ln(\cos x)$

(e) $m(x) = \int_0^{x^2} \sin^2 t dt$

(f) $n(x) = e^{-x}$

30. Compute the average value of $\sqrt{x}$ over the interval $[0, 2]$.

31. What does a definite integral represent? What does an indefinite integral represent?

32. Compute the following integrals. For a definite integral give an exact answer; points will be taken off for approximations.

(a) $\int_{-3}^{3} \sqrt{9 - x^2} \mathrm{d}x$

(b) $\int e^x \mathrm{d}x$

(c) $\int \cosh x \mathrm{d}x$

(d) $\int \frac{x - 4}{x^2} \mathrm{d}x$

(e) $\int \frac{5}{1 + x^2} \mathrm{d}x$

(f) $\int \frac{2x}{\sqrt{1 - x^2}} \mathrm{d}x$ [Hint: use substitution.]

(g) $\int (x + 1)e^{x^2+2x} \mathrm{d}x$ [Hint: use substitution.]

33. Find the point on the curve $y = \frac{-4}{x}$ which is the nearest to the origin $(0, 0)$.

34. The radius of a sphere is increasing at the uniform rate of 0.3 inches per second. At the instant when the surface area S becomes 100π square inches, what is the rate of increase, in cubic inches per second, of the volume V. [$S = 4\pi r^2$ and $V = \frac{4}{3}\pi r^3$.]

35. Compute the following limits.

(a)

$$\lim_{x\to\infty} \frac{2x^4 - x - 1}{7x^4 + 5x^2 - 1}$$

(b)

$$\lim_{x\to\infty} \frac{x^3 - x - 1}{x^6 + x^2 + 6}$$

(c)

$$\lim_{x\to-\infty} \frac{x^4 - x^2 + 2}{x^3 + x^2 + 1}$$

(d)

$$\lim_{x\to 0} \frac{x^2 - x + 5}{x^3 + 2x + 10}$$

(e)

$$\lim_{x\to 0} \frac{\sin x}{x}$$

36. Demonstrate how starting from that acceleration due to gravity is $a(t) = -32$ ft/s we compute the function which gives the height of a projectile at time t. Use that the initial height is $h(0) = h_0$ and the object was thrown with initial velocity $v(t) = v_0$.

Bonus Consider the following function.

$$\Lambda(x) = \int_0^x \frac{1}{t} dt.$$

Let $J(x)$ be the inverse function. Show the following.

1. $\Lambda(1) = 0$.
2. $\Lambda(ax) = \Lambda(a) + \Lambda(x)$.
3. $J(0) = 1$.
4. $J(x) = J'(x)$.
5. How do we use Λ to define the number e?

Bonus Use the formula

$$\sum_{i=1}^{n} i^5 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

to compute $\int_0^1 x^4 dx$. Recall this means for each positive integer n, partition the interval $[0, 1]$ into n equal parts. Each part is of width Δx. Compute the Right Riemann Sum. Then compute the limit as n goes to infinity.