Name: _____

Exam # 4 – Math 2311H – Spring 2014

1. $\displaystyle\int_0^8 \frac{dx}{\sqrt{1+x}} =$

   (A) 1 (B) $\frac{3}{2}$ (C) 2 (D) 4 (E) 6

2. Which of the following is an anti-derivative of $f(x) = \tan x$?

   (A) $\frac{1}{2}\tan^2 x + C$ (B) $\sec^2 x + C$ (C) $\sec x \tan x$

   (D) $\ln|\cos x| + C$ (E) $\ln|\sec x| + C$

3. What is the average value of the function $f(x) = 3t^3 - t^2$ over $[-1, 2]$?

   (A) $\frac{11}{4}$ (B) $\frac{7}{2}$ (C) 8 (D) $\frac{33}{4}$ (E) 16

4. At $t = 0$ a particle starts at rest and moves along the x-axis in such a way that at time t its acceleration is $24t^2$ ft/s. Through how many feet does the particle move during the first 2 seconds?

   (A) 32 (B) 48 (C) 64 (D) 96 (E) 192

5. $\displaystyle\int \frac{x^2}{e^{x^3}}\, dx =$

   (A) $-\frac{1}{3}\ln e^{x^3} + C$ (B) $-\frac{e^{x^3}}{3} + C$ (C) $-\frac{1}{3e^{x^3}} + C$

   (D) $\frac{1}{3}\ln e^{x^3} + C$ (E) $\frac{x^3}{3e^{x^3}} + C$

6. $\displaystyle\int \sin(2x+3)\, dx =$

   (A) $\frac{1}{2}\cos(2x+3) + C$ (B) $\cos(2x+3) + C$ (C) $-\cos(2x+3) + C$

   (D) $-\frac{1}{2}\cos(2x+3) + C$ (E) $-\frac{1}{5}\cos(2x+3) + C$

7. $\int (x^3 - 3x)dx =$

(A) $3x^2 - 3 + C$ (B) $4x^4 - 6x^2 + C$ (C) $\frac{x^4}{3} - 3x^2 + C$

(D) $\frac{x^4}{4} - 3x + C$ (E) $\frac{x^4}{4} - \frac{3x^2}{2} + C$

8. The slope of the line tangent to the graph of $y = \ln(x^2)$ at $x = e^2$

(A) $\frac{1}{e^2}$ (B) $\frac{2}{e^2}$ (C) $\frac{4}{e^2}$ (D) $\frac{1}{e^4}$ (E) $\frac{4}{e^4}$

9. The acceleration of a particle moving on the x-axis is given in terms of time t by $a(t) = 8 - 6t$. If the velocity of the body is 25 at time $t = 1$, and if $s(t)$ is the position of the particle at time t, what is $s(4) - s(2)$?

(A) 20 (B) 24 (C) 28 (D) 32 (E) 42

10. $\int_1^2 \frac{x-4}{x^2} dx =$

(A) $-\frac{1}{2}$ (B) $\ln 2 - 2$ (C) $\ln 2$ (D) 2 (E) $\ln 2 + 2$

11. Which of the following are true?

I. The average value of a positive function f on $[a, b]$ always lies between $f(a)$ and $f(b)$.

II.If $b > a$, then $\frac{d}{dx} \int_a^b e^{x^2}$ is positive.

III. If f is positive and differentiable on $[a, b]$, then $\int_a^b \frac{f'(x)}{f(x)} dx = \ln\left(\frac{f(b)}{f(a)}\right)$.

(A) I. and II. only (B) III. only (C) II. and III. only

(D) I., II., and III. (E) none of these

12. If $\int_{-1}^1 e^{-x^2} dx = k$, then $\int_{-1}^0 e^{-x^2} dx =$

(A) $-2k$ (B) $-k$ (C) $-\frac{k}{2}$ (D) $\frac{k}{2}$ (E) $2k$

13. Let $f(x) = xe^{-x}$.

(a) Derive $f'(x)$. Use this to find:

i. any critical points of $f(x)$.

ii. where $f(x)$ is increasing/decreasing.

iii. any relative extrema of $f(x)$.

(b) Derive $f''(x)$. Use this to find:

i. the concavity of $f(x)$.

ii. any inflection points of $f(x)$.

(c) Graph $f(x)$. Include any x-intercepts and y-intercepts.

(d) Are there any absolute extrema?

(e) Does $f(x)$ have any horizontal asymptotes?

14. Let $f(x) = \dfrac{x}{\sqrt{1+2x^2}}$.

(a) Use substitution to find $\displaystyle\int \frac{x}{\sqrt{1+2x^2}}dx$.

(b) Use your answer in (a) to find $\displaystyle\int_0^2 \frac{x}{\sqrt{1+2x^2}}dx$.

15. Let $f(x) = (x+1)\sqrt{2-x}$.

(a) Use substitution to find $\displaystyle\int (x+1)\sqrt{2-x}dx$. [Hint: (change of variable) use $u = 2-x$ and solve for x.]

(b) Find the average value of $f(x)$ over $[0, 2]$.