Name: _____

Exam # 3 – Math 2311H – Spring 2014

Throughout LRS denotes a left Riemann sum, RRS denotes a right Riemann sum, MRS denotes a midpoint Riemann sum, and TRS denotes a Trapezoidal Riemann Sum.

1. For what value of k will $x + \frac{k}{x}$ have a relative maximum at $x = -2$?

   (A) -4 (B) -2 (C) 2 (D) 4 (E) 0

2. When the area (in square units) of an expanding circle is increasing twice as fast its radius (in units), the radius is

   (A) $\frac{1}{4\pi}$ (B) $\frac{1}{4}$ (C) $\frac{1}{\pi}$ (D) 1 (E) π

3. The function $f(x) = 5x^4 - x^5$ has a point (or points) of inflection at which values of x?

   (A) $x = 0$ (B) $x = 3$ (C) $x = 0$ and $x = 3$ (D) $x = 0$ and $x = 4$ (E) $x = 4$

4. $\int_1^2 x^{-3} dx =$

   (A) $-\frac{7}{8}$ (B) $-\frac{3}{4}$ (C) $\frac{15}{64}$ (D) $\frac{3}{8}$ (E) $\frac{15}{16}$

5. $\int \sec^2 x \, dx =$

   (A) $\tan x + C$ (B) $\csc^2 x + C$ (C) $\cos^2 x + C$

   (D) $\frac{\sec^3 x}{3} + C$ (E) $2\sec^2 x \tan x + C$

6. If F and f are continuous functions such that $F'(x) = f(x)$ for all x, then $\int_a^b f(x)dx =$

   (A) $F'(a) - F'(b)$ (B) $F'(b) - F'(a)$ (C) $F(a) - F(b)$

   (D) $F(b) - F(a)$ (E) none of these

7. Which of the following statements are true?

   I. If $F'(x) = G'(x)$ on the interval $[a, b]$, then $F(b) - F(a) = G(b) - G(a)$.

   II. The value of $\int_2^2 \sin(x^2)dx$ is 0.

   III. If $f''(c) = 0$, then f has a point of inflection at c.

   (A) I. and II. only (B) III. only (C) II. and III. only

   (D) I., II., and III. (E) none of these

8. $\int_0^2 \sqrt{4 - x^2}dx =$

   (A) $\frac{3927}{1250}$ (B) $\frac{16}{3}$ (C) π (D) 2π (E) 4π

x	0	0.5	1.0	1.5	2.0
$f(x)$	13	4	5	8	3

9. A table of values for a continuous function f is shown above. If four equal subintervals of $[0, 2]$ are used, which of the following is the Right Riemann Sum approximation of $\int_0^2 f(x)dx$

   (A) 8 (B) 10 (C) 15 (D) 25 (E) 32

10. At a certain time the volume of a cube is increasing at 24 cubic inches per minute, and each edge is of the cube is increasing at 2 inches per minute. What is the length of each edge of the cube at this time?

    (A) 2in. (B) $2\sqrt{2}$in. (C) $\sqrt[3]{12}$ in. (D) 4in. (E) 8 in.

11. If g is a differentiable function such that $g(x) < 0$ for all x, and if $f'(x) = (x^2 - 9)g(x)$, which of the following is true.

(A) f has a relative maximum at $x = -3$ and a relative minimum at $x = 3$.

(B) f has a relative minimum at $x = -3$ and a relative maximum at $x = 3$.

(C) f has a relative minima at $x = -3$ and $x = 3$.

(D) f has a relative maxima at $x = -3$ and $x = 3$.

(E) It cannot be determined if f has any relative extrema.

12. A particle is moving clockwise around the unit circle (circle of radius 1 centered at the origin). At the point $(.6, .8)$ the particle has horizontal velocity $dx/dt = 3$. What is its vertical velocity dy/dt at this point?

(A) -3.875 (B) -3.75 (C) -2.25 (D) 3.75 (E) 3.875

13. In this problem we will calculate the integral $\int_0^2 x^3 dx$ using a Right Riemann Sum.

(a) Let $n = 5$. Split the interval into n equal parts. What is the length of each equal part: $\Delta x =$?

(b) Find the endpoints of each subinterval: $a_0, a_1, a_2, a_3, a_4, a_5$.

(c) Find the Right Riemann Sum: $R = \sum_{i=1}^{n} f(a_i)\Delta x$.

Next we find the total area.

(d) For a fixed n what is Δx? For each $i = 1, 2, \ldots, n$ find $a_i =$ and $f(a_i)$.

(e) Find the Right Riemann Sum (in terms of n): $R(n) = \sum_{i=1}^{n} f(a_i)\Delta x$.

(f) Use that $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$ to convert your answer in (e).

(g) Find $\int_0^2 x^3 dx = \lim_{n\to\infty} R(n)$.

14. An observer 70 meters south of a raiload crossing watches an eastbound train traveling at 60meters per second. At how many meters per second is the train moving away from the observer 4 seconds after it passes through the intersection?

15. Let $f(x) = \dfrac{1}{x^2+1}$. Find $f'(x)$ and $f''(x)$. Graph the function. Use all three of these to find the following:

    (a) critical points of $f(x)$,
    (b) where $f(x)$ is increasing/decreasing,
    (c) relative extrema
    (d) concavity
    (e) inflection points
    (f) limits at infinity
    (g) absolute extrema