**Name:** ____________________________

## Exam # 3 – Math 2311H – Fall 2013

1. If $f$ is a positive, continuous function on the interval $[a, b]$, which of the following rectangular approximation ethods has a limit equal to the actual area under the curve from $a$ to $b$ as the number of rectangles approaches infinty.

   I. Left Riemann Sum

   II. Right Riemann Sum

   III. Upper Sum

   (a) I. and II. only (b) III. only (c) I. and III. only
   (d) I., II., and III. (e) none of these

2. Which of the following statements are true?

   I. If $\int_a^b f(x)dx > 0$, then $f(x) \geq 0$ for all $x \in [a, b]$.

   II. If $f'(x) > 0$ for all real numbers $x$, then $f(x)$ increases without bound.

   III. If $f$ is a positive, increasing continuous function on $[a, b]$, then a left Riemann sum gives an under-approximation of $\int_a^b f(x)dx$.

   (a) I. and II. only (b) III. only (c) I. and III. only

   (d) I., II., and III. (e) none of these

3. If $\int_2^5 f(x)dx = 18$, then $\int_2^5 (f(x) + 4)dx =$

   (a) 20 (b) 22 (c) 23 (d) 25 (e) 30

4. Which of the following best approximates the average value of $f(x) = \cos x$ on the interval $[1, 5]$?

   (a) -0.990 (b) -.0450 (c) -0.128 (d) 0.412 (e) 0.998

5. $\int_1^2 x^{-3}dx =$

   (a) $-\frac{7}{8}$ (b) $-\frac{3}{4}$ (c) $\frac{15}{64}$ (d) $\frac{3}{8}$ (e) $\frac{15}{16}$

6. $\int \sec^2 x \; dx =$

(a) $\tan x + C$ (b) $\csc^2 x + C$ (c) $\cos^2 x + C$

(d) $\dfrac{\sec^3 x}{3} + C$ (e) $2\sec^2 x \tan x + C$

7. The volume of a cylindrical tin can with a top and bottom is to be $16\pi \text{in}^3$. If a minimum amount of tin is to be used to construct the can, what must be the height, in inches, of the can?

(a) $2\sqrt[3]{2}$ (b) $2\sqrt{2}$ (c) $2\sqrt[3]{4}$ (d)4 (e) 8

8. Which of the following statements are true?

I. If $F'(x) = G'(x)$ on the interval $[a, b]$, then $F(b) - F(a) = G(b) - G(a)$.

II. The value of $\int_2^2 \sin(x^2) dx$ is 0.

III. $\int_a^b f(x)g(x)dx = \int_a^b f(x)dx \int_a^b g(x)dx$.

(a) I. and II. only (b) III. only (c) I. and III. only

(d) I., II., and III. (e) none of these

9. Which of the following statements are true?

I. When $n \neq -1$, $\int x^n dx = \dfrac{x^{n+1}}{n+1} + C$.

II. $\int \sin x dx = \cos x + C$.

III. $\int \cos x dx = \sin x + C$.

(a) I. and II. only (b) III. only (c) I. and III. only

(d) I., II., and III. (e) none of these

10. If the average value of the function $f$ on the interval $[a, b]$ is 10, then $\int_a^b f(x)dx =$

(a) $\dfrac{10}{b-a}$ (b) $\dfrac{f(a)+f(b)}{10}$ (c) $10b - 10a$ (d) $\dfrac{b-a}{10}$ (e) $\dfrac{f(b)+f(a)}{20}$

11. $\int_0^2 \sqrt{4-x^2}dx =$

(a) $\frac{8}{3}$ (b) $\frac{16}{3}$ (c) $\pi$ (d) $2\pi$ (e) $4\pi$

12. The acceleration of a particle moving along the $x$-axis at time $t$ is given by $a(t) = 6t - 2$. If the velocity is 25 when $t = 3$ and the position is 10 when $t = 1$, then the position $x(t) =$

(a) $9t^2 + 1$ (b) $3t^2 - 2t + 4$ (c) $t^3 - t^2 + 4t + 6$
(d) $t^3 - t^2 + 9t - 20$ (e) $36t^3 - 4t^2 - 77t + 55$

13. $\int_0^x \sin t dt =$

(a) $\sin x$ (b) $-\cos x$ (c) $\cos x$ (d) $\cos x - 1$ (e) $1 - \cos x$

14. If $F(x) = \int_0^x \sqrt{t^3+1}dt$, then $F'(2) =$

(a) -3 (b) -2 (c) 3 (d) 4 (e) 18

For problems 15. and 16. use the following table.

| $x$ | 0 | 0.5 | 1.0 | 1.5 | 2.0 |
|---|---|---|---|---|---|
| $f(x)$ | 3 | 4 | 5 | 8 | 13 |

15. A table of values for a continuous function $f$ is shown above. If four equal subintervals of $[0, 2]$ are used, which of the following is the Right Riemann Sum approximation of $\int_0^2 f(x)dx$

(a) 8 (b) 10 (c) 15 (d) 25 (e) 32

16. A table of values for a continuous function $f$ is shown above. If four equal subintervals of $[0, 2]$ are used, which of the following is the trapezoidal approximation of $\int_0^2 f(x)dx$

(a) 8 (b) 10 (c) 15 (d) 25 (e) 32

17. (Extra Credit)

The graph of $y = f(x)$ is shown in the figure above. If $A_1$ and $A_2$ are positive numbers that represent the areas of the shaded regions, then in terms of $A_1$ and $A_2$,
$\int_{-4}^{4} f(x)dx - 2\int_{-1}^{4} f(x)dx =$

(a) $A_1$ (b) $A_1 - A_2$ (c) $2A_1 - A_2$ (d) $A_1 + A_2$ (e) $A_1 + 2A_2$

18. (Extra Credit)

The graph of $f(x)$ is shown in the figure above. If $g(x) = \int_a^x f(t)dt$, for what value of $x$ does $g(x)$ have a maximum?

(a) $a$ (b) $b$ (c) $c$ (d) $d$ (e) Not enough information.

19. (Extra Credit) The closed interval $[a, b]$ is partitioned into $n$ equal subintervals, each of which $\Delta x$, by the numbers $a_0, a_1, \ldots, a_n$ where $a = a_0 < a_1 < a_2 < \ldots < a_{n-1} < a_n = b$. What is $\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{a_i}\Delta x$?

(a) $\frac{2}{3}(b^{\frac{3}{2}} - a^{\frac{3}{2}})$ (b) $b^{\frac{3}{2}} - a^{\frac{3}{2}}$ (c) $\frac{3}{2}(b^{\frac{3}{2}} - a^{\frac{3}{2}})$

(d) $b^{\frac{1}{2}} - a^{\frac{1}{2}}$ (e) $2(b^{\frac{1}{2}} - a^{\frac{1}{2}})$

20. In this problem we will calculate the integral $\int_0^2 x^2 dx$ using Right Riemann Approximation.

(a) Let $n$ be a fixed natural number. Split the interval into $n$ equal parts. What is the length of each equal part: $\Delta x =?$

(b) Find the endpoints of each subinterval: $a_0, a_1, a_2, \ldots, a_n$. What is $a_i =?$

(c) Find the Right Riemann Approximation (in terms of $n$): $L(n) = \sum_{i=1}^{n} f(a_i)\Delta x$.

(d) Use that $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ to convert your answer in (c).

(e) Find $\int_0^2 x^2 dx = \lim_{n\to\infty} L(n)$.

21. Find two positive numbers that satisfy: the product is 147 and the sum of the first nmber plus three times the second number is a minimum.

22. Find the point on the curve $f(x) = x^2$ that is closest to the point $(1, \frac{1}{2})$.

23. Use an algebraic method to find the limit: (Be sure to briefly explain what you are doing.)

$$\lim_{x \to -\infty} \frac{\sqrt{x^2 + x}}{-2x}$$

24. An object is thrown vertically with an initial velocity of $v_0$ and an initial height of $h_0$. Using that the acceleration on the object due to gravity is $-32 ft/s^2$

   (a) work through and show that the height of the function at time $t$ (in seconds) is given by

$$h(t) = -16t^2 + v_0 t + h_0.$$

   (b) Use (a) to find the following:

      i. the time at which it reaches its maximum height.
      ii. the maximum height.
      iii. the time at which it hits the ground.
      iv. the velocity of the object when it hits the ground.