Name: _____

Exam # 2 – Math 2311H – Spring 2014

Directions. For the free response questions make sure to show work or explain how you obtained your answer.

1. What is $\lim_{\Delta x \to 0} \dfrac{8(\frac{1}{2}+\Delta x)^8 - 8(\frac{1}{2})^8}{\Delta x}$?

   (A) 0 (B) $\frac{1}{2}$ (C) 1 (D) -1 (E) does not exist

2. The Mean Value Theorem guarantees the existence of a special point on the graph of $y = \sqrt{x}$ over the interval $[0, 4]$. What is the x-value of the coordinate of this point?

   (A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) 4

3. If $h(x) = f^2(x) - g^2(x)$, $f'(x) = -g(x)$ and $g'(x) = f(x)$, then $h'(x) =$

   (A) 0 (B) 1 (C) $-4f(x)g(x)$ (D) $(-g(x))^2 - (f(x))^2$ (E) $-2(-g(x)+f(x))$

4. If $f(x) = x + \sin x$, then $f'(x) =$

   (A) $1 + \cos x$ (B) $1 - \cos x$ (C) $\cos x$ (D) $\sin x - x\cos x$ (E) $\in x + x\cos x$

5. If $f(x) = \dfrac{x-1}{x+1}$ for all $x \neq -1$, then $f'(1) =$

   (A) -1 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 1

6. If $\tan(xy) = x$, then $\dfrac{dy}{dx} =$

   (A) $\dfrac{1 - y\tan(xy)\sec(xy)}{x\tan(xy)\sec(xy)}$ (B) $\dfrac{\sec^(xy) - y}{x}$ (C) $\cos^2(xy)$

   (D) $\dfrac{\cos^2(xy)}{x}$ (E) $\dfrac{\cos^2(xy) - y}{x}$

7. If $f(x) = x + \dfrac{1}{x}$, then the set of values for which f increases is

(A) $(-\infty, -1) \cup (1, \infty)$ (B) $(-1, 1)$ (C) $(-\infty, \infty)$

(D) $(0, \infty)$ (E) $(-\infty, 0) \cup (0, \infty)$

8. If $x^2 + xy + y^3 = 0$, then $\dfrac{dy}{dx} =$

(A) $-\dfrac{2x+y}{x+3y^2}$ (B) $-\dfrac{x+3y^2}{2x+y}$ (C) $\dfrac{-2x}{1+3y^2}$ (D) $\dfrac{-2x}{x+3y^2}$ (E) $-\dfrac{2x+y}{x+3y^2-1}$

9. The function defined by $f(x) = x^3 - 3x^2$, for all real numbers x, has a relative maximum at $x =$

(A) -2 (B) 0 (C) 1 (D) 2 (E) 4

10. An equation of the line tangent to the graph of $f(x) = x(1-2x)^3$ at the point $(1, -1)$ is

(A) $y = -7x + 6$
(B) $y = -6x + 5$
(C) $y = -2x + 1$
(D) $y = 2x - 3$
(E) $y = 7x - 8$

11. If u, v and w are nonzero differentiable functions the the derivative of $\dfrac{uv}{w}$ is

(A) $\dfrac{uv' + u'v}{w'}$ (B) $\dfrac{u'v'w - uvw'}{w^2}$ (C) $\dfrac{uvw' - u'vw - u'vw}{w^2}$

(D) $\dfrac{u'vw + uv'w + uvw'}{w^2}$ (E) $\dfrac{uv'w + u'vw - uvw'}{w^2}$

12. The graph of the derivative of f is shown above. Which of the following could be the graph of f?

13. Consider the function

$$f(x) = \begin{cases} x^2 & x < 3; \\ 6x - 9 & x \geq 3. \end{cases}$$

At $x = 3$, $f(x)$ is

(A) undefined.

(B) continuous but not differentiable.

(C) differentiable but not continuous.

(D) neither continuous nor differentiable.

(E) both continuous and differentiable.

Remember to show work or explain your answers!!

14. Consider the function $f(x) = x^3 - 3x^2 + 12$ over the closed interval $[-2, 4]$.

    (a) Find all critical points of $f(x)$ in the interval $[-2, 4]$

    (b) Determine the absolute maximum value and absolute minimum value over the intervals.

    (c) Determine where the function is increasing. Determine where the function is decreasing.

    (d) Identify all relative extrema.

15. Compute the derivative of $f(x) = x^3$ as a limit of a difference quotient.

16. A person is on a diving board 32 ft from the surface of the water. The diver jumps into the air with an initial velocity of 16 ft/s.

    (a) Express the height of the diver as a quadratic equation.

    (b) At what time does the diver hit the surface of the water?

    (c) What is the maximum height of the diver?

    (d) What is the speed of the diver when entering the water?

Extra Credit Determine the points at which the graph of the function has a horizontal tangent line.

$$f(x) = \frac{x^2}{x-1}$$