Name: ______________________________

Exam # 2 – Math 2311H – Fall 2013

Directions. For the free response questions make sure to show work or explain how you obtained your answer.

1. The function f, given by $f(x) = x^3 + 12x - 24$, is

   (a) increasing for $x < -2$, decreasing for $-2 < x < 2$, increasing for $x > 2$

   (b) decreasing for $x < 0$, increasing for $x > 0$

   (c) increasing for all x

   (d) decreasing for all x

   (e) decreasing for $x < -2$, increasing for $-2 < x < 2$, decreasing for $x > 2$

2. Suppose x and y are implicitly defined in terms of t so that

$$\tan y + x^3 = y^2 + 1.$$

   If $\dfrac{dx}{dt} = -2$ what is the value of $\dfrac{dy}{dt}$ at the point $(1, 0)$?

   (a) -6 (b) -2.5 (c) 0 (d) π (e) 6

3. What is the slope of the line tangent to the curve $x^2 + 2xy + 3y^2 = 2$ when $y = 1$.

   (a) $\dfrac{-1}{2}$ (b) $\dfrac{-1}{8}$ (c) -1 (d) 0 (e) $\dfrac{1}{8}$

4. The function $f(x) = 4x^3 - 8x^2 + 1$ on the interval $[-1, 1]$ has an absolute minimum at $x =$?

   (a) -11 (b) -1 (c) 0 (d) (e) $\dfrac{4}{3}$

5. A 13-foot ladder is sliding down a wall at a rate of -4 ft/s. When the top of the ladder is 5ft from the ground, how fast is the foot of the ladder moving away from the wall (in ft/s).

   (a) $\dfrac{5}{3}$ (b) $\dfrac{8}{3}$ (c) 4 (d) 8 (e) $\dfrac{3}{8}$

6. If $f'(x) = g(x)$ and $h(x) = \sin x$, then $(f \circ h)'(x)$ equals?

   (a) $g(\sin x)$ (b) $3\cos x \cdot g(x)$ (c) $g'(x)$ (d) $\cos x \cdot g(\sin x)$ (e) $\sin x \cdot g(\sin x)$

7. Let $g(x) = x^2 \sin^2 x$. Then $g'(x) =$

   (a) $4x \sin x \cos x$

   (b) $2x \cos^2 x$

   (c) $2x \sin^2 x + x^2 \cos^2 x$

   (d) $2x \sin^2 x + 2x^2 \sin x \cos x$

   (e) $2x \sin x \cos x$

8. If $y = \dfrac{3}{4+x^2}$, then $\dfrac{dy}{dx} =$

   (a) $\dfrac{-6x}{(4+x^2)^2}$ (b) $\dfrac{3x}{4+x^2}$ (c) $\dfrac{6x}{(4+x^2)^2}$ (d) $\dfrac{-3}{(4+x^2)^2}$ (e) $\dfrac{3}{2x}$

9. The function defined by $f(x) = x^3 - 3x^2$ (for all reals) has a relative maximum at $x =$

   (a) -2 (b) 0 (c) 1 (d) 2 (e) 4

10. The volume of a cone of radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$. If the radius and the height both increase at a constant rate of $\frac{1}{2}$ cm/s, at what rate, in cubic cm/s, is the volume increasing when the height is 9cm and the radius is 6cm?

    (a) $\frac{1}{2}\pi$ (b) 10π (c) 24π (d) 54π (e) 108π.

11. An equation of the line tangent to $f(x) = x^3 + 3x^2 + 2$ at its point of inflection is

    (a) $y = -6x - 6$ (b) $y = 3x + 1$ (c) $y = -3x + 1$

    (d) $y = 4x + 1$ (e) $y = 2x + 10$

12. Determine the points at which the graph of the function has a horizontal tangent line.

$$f(x) = \frac{x^2}{x-1}$$

    (a) $x = 1, 2$ (b) $x = 1$ (c) $x = 2$ (d) $x = 0, 2$ (e) $x = 0, 1, 2$

13. The area of a circular region is increasing at a rate of $60\pi\ m^2/s$. When the area of the region is $36\pi\ m^2$, how fast, in m/s, is the radius of the region increasing?

    (a) 5 (b) 8 (c) 25 (d) $5\sqrt{3}$ (e) $15\sqrt{3}$

14. The graph of $y = \dfrac{-5}{x-2}$ is concave down for all values x such that

(a) $x < 0$ (b) $x < 2$ (c) $x < 5$ (d) $x > 0$ (e) $x > 2$

15.

16.

17.

18.

Remember to show work or explain your answers!!

19. Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$. The derivative of h is given by $h'(x) = \dfrac{x^2 - 2}{x}$ for all $x \neq 0$.

(a) Find the critical values of h.

(b) Find all values of x for which the graph of h has a horizontal tangent line, and determine whether h has a local maximum, a local minimum, or neither at each of the points.

(c) On what intervals, if any, is the graph of h concave up?

(d) Write an equation of the line tangent to the graph of h at $x = 4$. [Hint: use that $h(4) = -3$.]

(e) Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$?

20. For this problem consider $f(x) = \sin x + \cos x$ over the interval $[0, 2\pi]$.

(a) Find $f'(x)$ and $f''(x)$.

(b) Find all critical points of $f(x)$ in the interval $[0, 2\pi]$

(c) Find the open intervals on which the function is increasing or decreasing.

(d) Apply the First Derivative Test to identify all relative extrema. What is the absolute maximum and minimum values over the interval $[0, 2\pi]$.

(e) Determine all inflection points, if any.

21. All edges of a cube are expanding at a rate of 6cm per second. How fast is the volume changing when each edge is 10cm?

22. A coffeepot has the shape of a cylinder with radius 5inches, as shown in the figure. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t, measured in second. The volume V of coffee in the pot is changing at the rate $-25\pi h^2$ cubic inches per second. (The volume V of the cylinder with radius r and height h is $V = \pi r^2 h$.)

(a) In this problem what is $\dfrac{dr}{dt}$?

(b) Show that $\dfrac{dh}{dt} = -h^2$.